ICT4Depression

User-friendly ICT Tools to Enhance Self-Management and Effective Treatment of Depression in the EU

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Deliverable 3.1: Report addressing formal specification of abstract ontology on therapy progress as well as mapping between the interpreted information coming from sensor and the abstract ontology

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PU	Public	X
PP	Restricted to other programme participants (including the Commission Services).	
RE	Restricted to a group specified by the consortium (including the Commission Services).	
CO	Confidential, only for members of the consortium (including the Commission Services).	

Executive Summary

This deliverable presents a specification of the ontology used to express information obtained from the patient within the ICT4Depression system. These measurements are based on the extensive use-cases described in D4.1 and the therapeutic intervention descriptions as given in D1.2. In addition, the deliverable presents approaches to abstract from these very precise measurements that are performed frequently, and look at the general trends of the patient's wellbeing as well as the therapeutic involvement. In order to make this abstraction, a three layered approach is proposed. On the first level, the low-level measurements are placed. The second layer encompasses the identification of trends within the low-level measurements over time (i.e. abstraction over time). Finally, on the third layer these trends are coupled to the overall picture of the patient (how well is the patient doing, and how is the therapeutic involvement). In order to connect the various layers, a formal logical approach is used that is able to handle qualitative as well as quantitative aspects.

Contents

1. Introduction

In this document, the ontologies that can be used to express the progress of the patient as well as the current state of the patient are described. Hereby, different types of ontologies are identified, ranging from relatively low level expression about the patient (e.g. the patient is currently walking) to more general trends of the patient (e.g. the therapy seems to be a success, the homework is performed on a regular basis and the patient is feeling better). All of these ontologies have been based upon the information obtained from deliverables D4.1 (consisting of an extensive number of use-cases) and D1.2 (containing a detailed description of the therapeutic modules). Furthermore, in this deliverable a formal approach is explained which allows for the expression of relations between the lower level expression of the patient and the more general level trends, thereby taking temporal issues under consideration. The precise relationships are expressed using this language as well. Note that this document describes a large set of possible measurements. Within the ICT4Depression system, a subset of the measurements considered here will be used. In order to be able to cope with this, the approach has been set up in a modular fashion.

2. Ontological terms for a patient

In order to describe the current state of the patient, three different levels are distinguished within the ontology. Figure 1 below shows the three ontologies, and their relationship with the reasoning engine, which will be detailed in a later deliverable.

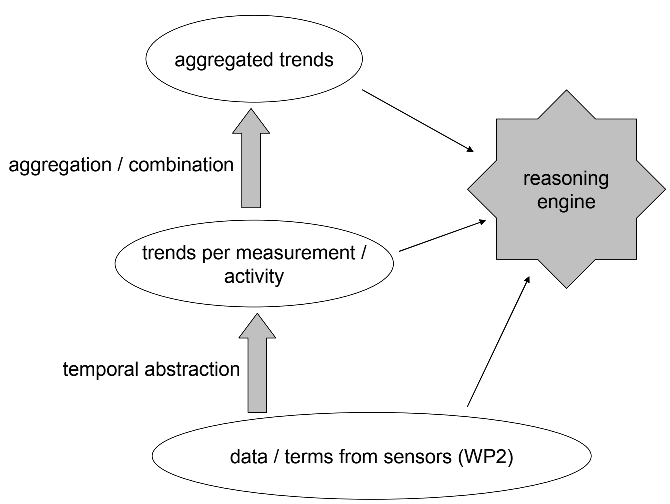

Figure 1. Different ontologies and their relationship with the reasoning engine.

The first ontology consists of the terms that are used to describe the current data that comes from the sensors. This could for example include data about what questionnaires have just been filled in by the patients, or the heart rate. One level higher, more abstract trends are measured per activity which try to look at trends over certain periods of time. For instance, the fact that the heart rate is continuously increasing, or continuously high. In the final part, information about these trends from multiple measurements or activities are combined, thereby resulting in a good view on the general status of the patient. Below, the content of each of these ontologies are explained in more detail. Note that here an informal description of the various elements in the ontology is presented. The precise formal specification of the ontology as well as the abstraction is expressed in Section 4 of this document.

2.1. Current Patient State

In order to express the current state of the patient, the following construct is used:

state(patient, t) |= patient_data(device, data, probability)

In this specification, the state(patient, t) represents the state of the sensor devices of the indicated patient at the indicated time point. The right hand side of the term (patient_data(device, data, probability)) represents the actual knowledge which has been obtained about the patient via the sensors. Hereby, 'device' expresses the source of the information, 'data' the particular element that has been measured, and 'probability' expresses the certainty of the measurement. The symbol |= represents the satisfaction relation, see Section 3 for a more elaborate discussion. Instead of using such a symbol, it is also possible to use a predicate of the form at(t, patient, patient_data(device, data, probability). Below, an overview is given of the elements in the data. These elements have been divided into three parts: (1) information about the current (both physical and mental) state of the patient; (2) information about the general measurements performed within the ICT4Depression support system, and (3) specific measurements for therapies. Each of these measurements is related to the actual devices that are able to measure this information. In this case, the mobile phone and the website are available for receiving explicit input from the patient (e.g. an answer to a question posed). Furthermore, other measurement devices include the sensors on the mobile phone (accelerometer, GPS, microphone), the measurements on the physiological device (heart rate - HR, heart rate variability – HRV, and the galvanic skin response - GSR), and the pill box.

2.1.1 Data elements: Mental and Physical Patient State.

The first type of data element discussed concerns the information about the physical and mental state of the patient. Hereto, a number of measurements are performed. The data elements are presented in Table 1 below. In the table, five columns are present. The first describes the state which is measured. In the second column, the type of data that is stored is shown. The frequency with which the data is measured is shown in the third column whereas the period for which the rating holds, is expressed in the fourth. Note that some elements will be measured using a generic questionnaire which is presented to the patient with a relatively low frequency. Finally, in the fifth column the device (or combination of devices) that perform the measurements is shown. The formal descriptions of the various measurements in the form of predicates will be shown in Section 4.1.

Table 1. Mental and physical states of the patient within the ICT4Depression system

Mental or physical state	Type of data	Frequency	Rating holds for period	Measurement device
Stress level	Rating, 1-10	Every 60 minutes	Current time point	HRV, GSR, Respiration
Mood level	Rating, 1-10	Depends on patient settings (default 5 times per day)	Current time point	Mobile phone
Activity level	Rating, 1-10	Every 60 minutes	Current time point	HR, mobile phone sensors
Social interaction	Rating 1-10	Every 60 minutes	Current time point	mobile phone sensors
Sleep quality	Rating 1-10	Every day	The past night	Mobile phone, website: questionnaire
Rating current therapy (how much does the patient still like the current therapy)	Rating 1-10	Depends on the progress of therapy	Current time point	Mobile phone, website
Anxiety level	Rating, 1-10	Every time the generic questionnaire is filled in	Period between current and previous questionnaire	Mobile phone, website: questionnaire
Positivity of thoughts	Rating, 1-10	Every time the generic questionnaire is filled in	Period between current and previous questionnaire	Mobile phone, website: questionnaire
Motivation	Rating 1-10	Every time the generic questionnaire is filled in	Current time point	Mobile phone, website: questionnaire

Self-efficacy	Rating 1-10	Every time the generic questionnaire is filled in	Current time point	Mobile phone, website: questionnaire
Number of GP visits	Number ≥ 0	Every time the generic questionnaire is filled in	Period between current and previous questionnaire	Mobile phone, website: questionnaire
Health expenses	Real number ≥ 0	Every time the generic questionnaire is filled in	Period between current and previous questionnaire	Mobile phone, website: questionnaire
Number of working hours	Real number ≥ 0	Every time the generic questionnaire is filled in	Period between current and previous questionnaire	Mobile phone, website: questionnaire
Prescribed Medication	Medicine type and frequency	Every time the generic questionnaire is filled in	Period between current and previous questionnaire	Mobile phone, website: questionnaire

2.1.2 Data elements: Generic Therapeutic Measurements.

Table 2 shows the measurements that are common among all the therapies within the ICT4Depression system, namely the scheduling of rating certain states of the patient, the reading of certain chapters consisting of information, certain homework that is performed (assignments which are part of a specific treatment module), and a generic questionnaire about the progress of the patient. In case the data presented below is just a fact without any parameters, the type of data is set to *Fact.*

Table 2. Generic therapeutic measurements within the ICT4Depression system

Therapeutic measurement	**Type of data**	**Frequency**	**Rating holds for period**	**Measurement device**
State rating scheduled[1]	Fact	Every time a state rating has been scheduled	The current time point	ICT4D database with therapy specification
State rating performed[1]	Fact	Every time a state rating has been performed	The current time point	Mobile phone
Homework deadline[2]	Fact	Every time a homework deadline is scheduled	The current time point	ICT4D database with therapy specification
Homework	Percentage	Every time a homework	The current time	Mobile phone,

[1] The state events concern each of the data elements in Table 1 that are not part of the questionnaire (i.e. do not have a frequency equal to "every time the questionnaire is filled in".

[2] The homework and chapter measurements all include a dedicated identifier expressing what specific part (i.e. chapter and homework) they concern.

submission: percentage done[2]	(% of questions answered)	assignment is submitted	point	website
Homework submission: time spent[2]	Time in minutes	Every time a homework assignment is submitted	The current time point	Mobile phone, website
Chapter deadline[2]	Fact	Every time a chapter deadline is scheduled	The current time point	ICT4D database with therapy specification
Chapter finished: percentage read[2]	Percentage (% of material read)	Every time a chapter within the therapy is completed	The current time point	Mobile phone, website
Chapter finished: time spent[2]	Time in minutes from activation of a Chapter till activation of following Chapter	Every time a chapter within the therapy is completed	The current time point	Mobile phone, website
Questionnaire deadline	Fact	Every time a questionnaire deadline is scheduled	The current time point	ICT4D database with therapy specification
Questionnaire finished: percentage answered	Percentage (% of questions answered)	Every time a questionnaire within the therapy is completed	The current time point	Mobile phone, website

2.1.3 Data elements: Therapy specific measures.

Finally, each module or part within the ICT4Depression system also consists of a number of specific measurements. These are treated per part/module.

Initial questionnaire. Within the initial questionnaire, the current state of the patient is assessed, including the preferences of the patient with respect to certain key elements within each of the therapeutic modules.

Table 3. Specific therapeutic measurements within the ICT4Depression system: Initial questionnaire

Initial questionnaire measurement	Type of data	Frequency	Rating holds for period	Measurement device
Therapies followed in the past	Type of therapy followed	When the patient starts with the ICT4Depression therapy	Period before the start of the therapy	Mobile phone, website
Patient's preference for performing	Rating, 1-10	When the patient starts with the ICT4Depression	Current time point	Mobile phone, website

activities		therapy		

Medicine intake module. Within the medicine intake module, measurements are performed to monitor the behavior of the patient with respect to medicine intake and also provide support to the patient. The specific measurements that are part of this module are shown in Table 4.

Table 4. Specific therapeutic measurements within the ICT4Depression system: Medicine intake module

Medicine intake module: measurements	**Type of data**	**Frequency**	**Rating holds for period**	**Measurement device**
Medicine intake scheduled	Medicine identifier	Every time a medicine intake has been scheduled according to the drug regime, stored in the adherence monitoring system	The current time point	Adherence monitoring system
Medicine intake performed	Medicine identifier	Every time a medicine has been taken from the pill box and the pill box is put back on the reader	The time point at which the medicine was taken	Adherence monitoring system (MEMS)

Behavioral activation module. An overview of the elements measured within the behavioral activation module is shown in Table 5. In the table the so-called therapeutic activities are activities the patient can select from within the therapy (and schedule for a particular time interval). These are commonly combinations of simple activities. For example, going out for dinner might be a therapeutic activity which has been selected by the patient. The list of activities under investigation is shown in Table 6 and is a list which is currently used in an Internet-based therapy. Potentially, patients can extend this list with their own preferred activities, but these will then simply not be part of the recognition process. The sensor recognizable activities are the activities that can be recognized by the devices, for instance to walk. These are shown in Table 7. Finally, also locations are recognized in order to facilitate a translation of the sensed activities into high-level activities. These are shown in Table 8. How these levels can be related to each other will be discussed in Section 4.2.

Table 5. Specific therapeutic measurements within the ICT4Depression system: Behavioral activation module

Behavioral activation module: measurements	**Type of data**	**Frequency**	**Rating holds for period**	**Measurement device**
Activity scheduled	Therapeutic	Every time an activity	The start time of	Mobile phone,

	activity, interval scheduled	has been scheduled by the patient	the scheduled activity	website
Activity performed (automated)	Sensor recognizable activity, interval	Every time an activity has been performed by the patient	The end time of the activity	Mobile phone sensors, HRV, HR, Respiration
Activity performed (manual)	Therapeutic activity, yes/no	Every time the patient is asked whether a certain activity has been performed	The interval of the activity	Mobile phone
Activity rating scheduled	Therapeutic activity	Every time an activity that has been scheduled by the patient has ended	The interval during which the activity was performed	Mobile phone, website
Activity rated	Therapeutic activity, interval of rated activity, rating 1-10	Every time the patient rates an activity	The interval during which the activity was scheduled	Mobile phone
Location event	Type of location, interval during which the patient is at the location	Every time it is recognized that the patient is at a pre-defined relevant location.	The interval during which the patient was at that location.	Mobile phone (GPS, Radios)

Table 6. Therapeutic activities considered (list provided by Psychologists and has been used in therapy before)

Activity
Smiling
Relaxing
Being close to happy people
Eat good food
Think about something good in the future
People show interest in what you say
Think that people think you are nice
Thinking about people you like
Enjoy nice scenery
Breathe clean air
Enjoy company of friends
Feeling calm and satisfied
Being found sexually attractive by someone else
Kissing
Watching other people

Having an open-minded conversation
Enjoy the sunshine
Wearing clean clothes
Having spare time
Executing a plan in your own way
Sleeping well at night
Enjoy good music
Having intimate contacts
Smiling at other people
Hearing from someone he/she loves you
Reading stories, novels, poems, or plays
Making a plan, or taking initiative to do something
Go out to a restaurant
Telling someone you love him/her
Making love
Being in the company of someone you love
Enjoy good times with the family
Giving someone a compliment
Taking a drink with friends
Meeting someone
Having the feeling that you’re good at something
Formulating something clearly
Having animals around you
Making a good impression within a group of people
Having a vivid conversation
Feeling the presence of God in your life
Making plans for a trip or holiday
Listening to the radio
Learning something new
Meeting old friends
Successfully finishing a job
Being asked for help or advice
Amusing others
Receiving a compliment or being told that you have done something right

Table 7. Sensor recognizable activities considered

Activity	**Description**
Walking	The patient is walking.
Sitting	The patient is sitting.
Running	The patient is running.
Cycling	The patient is cycling.
Climbing stairs	The patient is climbing stairs.
Social noise	The patient is in the vicinity of other people.

Table 8. Locations

Location	Description
Park	
Restaurant	
Gym	
Home	
Office	
Family	
GP office	

Cognitive restructuring. The number of additional information elements within the cognitive restructuring module is limited. Patients should identify their negative thoughts and learn to think in a more positive manner. The only additional element added is that the patients are asked to rate how well they have been able to identify their negative thoughts. This is shown in Table 9.

Table 9. Specific therapeutic measurements within the ICT4Depression system: Cognitive restructuring module

Cognitive restructuring: measurements	Type of data	Frequency	Rating holds for period	Measurement device
Thought registration event	Positive or negative thought	Every time a patient registers a negative/positive thought.	The period during which the patients should write down their thoughts (during exercise 1)	Mobile phone, website
Belief in negative automatic thoughts rated	Rating, 1-10	Every time the patient is asked for his belief in negative automated thoughts	Exercise 2 of the module	Mobile phone, website
Emotion strength rated	Rating, 1-10	Every time the patient is asked for his emotion strengths	Exercise 3 of the module	Mobile phone, website
Thought challenge event	Challenge to negative thought	Every time a patient challenges a negative thought.	The period during which the patients should challenge their thoughts (during exercise 4)	Mobile phone, website

Problem solving. Within problem solving, patients should identify their problems and try to come up with solutions. The information used in the ontology in this case involves when problems are identified and when they are at least partially solved. They are shown in Table 10.

Table 10. Specific therapeutic measurements within the ICT4Depression system: Problem solving module

Problem solving: measurements	**Type of data**	**Frequency**	**Rating holds for period**	**Measurement device**
Problem identified	Problem identification, the severity of the problem (important/unimportant)	Every time the patient is asked to identify a problem	The current time point	Mobile phone, website
Problem solved	Problem identification, Rating 1-10 (how well has the problem been solved)	Every time the patient has (partially) solved a problem	The current time point	Mobile phone, website

Exercise Therapy. Within exercise therapy the additional elements obtained are very similar to the ones previously indicated for Behavioral Activation (i.e. scheduling of exercises during certain time-intervals, seeing whether they have been performed, et cetera). Note that in these case only the low level activities are used (and additional exercises that might not be recognizable, together called *exercise*) as they precisely encompass the physical exercises being conducted within exercise therapy. Finally, the heart rate is also logged.

Table 11. Specific therapeutic measurements within the ICT4Depression system: Exercise module

Exercise therapy: measurements	**Type of data**	**Frequency**	**Rating holds for period**	**Measurement device**
Exercise scheduled	Exercise, interval scheduled	Every time an exercise has been scheduled by the patient	The start time of the scheduled exercise	Mobile phone, website
Exercise performed (automated)	Exercise, interval	Every time an exercise has been performed by the patient	The end time of the exercise	Mobile phone sensors, HR, Respiration
Exercise performed (manual)	Exercise, yes/no	Every time the patient is asked whether a certain exercise has been performed	The interval of the exercise	Mobile phone
Exercise rating scheduled	Exercise	Every time an exercise that has been scheduled by the patient has ended	The interval during which the exercise was performed	Mobile phone, website
Exercise rated	Exercise, interval of rated activity, rating 1-10	Every time the patient rates an activity	The interval during which the exercise was	Mobile phone

			scheduled, or just before the exercise was scheduled	
Heart rate	0-255	Every 5 minutes, every 1 minutes during exercise	Current time point	HR

Relapse Prevention. Finally, in the relapse prevention module, the patient is regularly asked to rate his/her mood. In case the mood is shown to degrade over time, prior modules can be repeated.

Table 12. Specific therapeutic measurements within the ICT4Depression system: Relapse prevention module

Relapse prevention: measurements	**Type of data**	**Frequency**	**Rating holds for period**	**Measurement device**
Problems	Problem identification, the severity of the problem (important/unimportant)	Every time the patient is asked to identify a problem	The period between the current and the previous rating	Mobile phone, website
General activity level (manual)[3]	Rating, 1-10	Every time the patient is asked to rate the general activity level	The period between the current and the previous rating	Mobile phone, website
The measurements for medicine adherence will also be included; the patients will continue to use their automated pill box.				

2.2. Patient State Trends

Next to the fact that particular events take place, more can be said when trends can be distinguished. Just from one individual missed mood rating not a lot can be said, particular structural problems can only be determined when looking at the trends over time. In this case, the following trends during particular periods are distinguished:

- **Increasing during a period x:** The general trend is that a particular aspect of the therapy or state of the patient is increasing during a certain time period x.
- **Decreasing during a period x:** The general trend is that a particular aspect of the therapy or state of the patient is decreasing during a certain time period x.

[3] Note that the activities are also measured automatically, see Table 1 "Activity Level".

- **Stable (fluctuations within certain boundaries) during a period x:** The general trend for a particular aspect of the therapy or the state of the patient is stable during a certain period x.
- **Average over a period x is above a threshold th.** The average value for a particular aspect of the therapy or the state of the patient is above a certain threshold value th during a certain time period x.
- **Average over a period x is below a threshold th.** The average value for a particular aspect of the therapy or the state of the patient is below a certain threshold value th during a certain time period x.
- **Percentage of cases above a threshold th during period x.** The percentage of measurements of a certain aspect of the therapy or state of the patient above a certain threshold th during a certain time period x.
- **Percentage of cases below a threshold th during period x.** The percentage of measurements of a certain aspect of the therapy or state of the patient below a certain threshold th during a certain time period x.

Formalizations of these general trends will be shown in Section 4.2.

2.2.1 Trends: Mental and Physical Patient State.

Below, an overview is given of the trends in the mental and physical state of the patient. Note that in this case only the states that are measured with a relatively high frequency are shown as these are the ones that can be used to create a good idea on how the patient is doing.

Table 13. Trends for mental and physical state

State	Trend	
Stress	Patient generally stressed	Percentage of stress measurements indicating a high stress level (i.e. the value is above a certain threshold h) during period x above a certain percentage value s_{high}
	Patient occasionally stressed	Percentage of stress measurements indicating a high stress level (i.e. the value is above a certain threshold h) during period x below a certain percentage value s_{high} but above a certain percentage value s_{low}
	Patient not stressed	Percentage of stress measurements indicating a high stress level (i.e. the value is above a certain threshold h) during period x below a certain percentage value s_{low}
Mood	Patient mood improving	Mood level increasing during a certain period x

	Patient mood stable	Mood level stable within certain bounds in period x
	Patient mood getting worse	Mood level decreasing during a certain period x
	Patient mood generally good	Percentage of mood measurements indicating a good mood level (i.e. the value is above a certain threshold h) during period x above a certain percentage value m_{happy}
	Patient mood generally bad	Percentage of mood measurements indicating a good mood level (i.e. the value is above a certain threshold h) during period x below a certain percentage value m_{happy}
Activity level	Patient becoming more active	Activity level increasing during a certain period x
	Patient activity level stable	Activity level stable within certain bounds in period x
	Patient becoming less active	Activity level decreasing during a certain period x
	Patient generally active	Percentage of activity measurements indicating a high activity level (i.e. the value is above a certain threshold h) during period x above a certain percentage value e_{fit}
	Patient generally inactive	Percentage of activity measurements indicating a high activity level (i.e. the value is above a certain threshold h) during period x below a certain percentage value e_{fit}
Social interaction	Patient socially becoming more active	Social interaction level increasing during a certain period x
	Patients socially stable	Social interaction level stable within certain bounds in period x
	Patient socially becoming less active	Social interaction level decreasing during a certain period x
	Patient generally socially active	Percentage of social activity measurements indicating a high social activity level (i.e. the value is above a certain threshold h) during period x above a certain percentage value si_{active}
	Patient generally socially inactive	Percentage of social activity measurements indicating a high social activity level (i.e. the value is above a certain threshold h) during period x below a certain percentage value si_{active}
Sleep quality	Patients sleep quality increasing	Sleep quality increasing during a certain period x
	Patients sleep quality stable	Sleep quality stable within certain bounds

		in period x
	Patient sleep quality decreasing	Sleep quality decreasing during a certain period x
	Patient generally sleeping with high quality	Percentage of sleep quality measurements indicating a high sleep quality (i.e. the value is above a certain threshold h) during period x above a certain percentage value sq_{good}
	Patient generally sleeping with low quality	Percentage of sleep quality measurements indicating a high sleep quality (i.e. the value is above a certain threshold h) during period x below a certain percentage value sq_{good}

The other states that are measured involve measurements on a less frequent basis, namely when a questionnaire is given to the patient. Since this is only planned once a month, trends in those rating will not add much to the overall knowledge in trends for the particular patients. As a result, trends for these states will currently not be considered.

2.2.2 Trends: Generic Therapeutic Measurements

The trends from a generic therapeutic perspective are discussed in this section and are shown in Table 14. Note that some of these trends require a combination of two measurements, namely at what time a certain therapeutic activity has been scheduled, and when it has actually been performed. A formalization of this combination will be presented in Section 4.2.

Table 14. Trends for general therapeutic measurements

State	Trend	
Ratings (whether they are performed on time or not)	State ratings good	Percentage of timely performed ratings (i.e. the time at which the rating has been performed does not deviate more than d from the prescribed time) during period x above a certain value r_{good}
	State ratings bad	Percentage of timely performed ratings (i.e. the time at which the rating has been performed does not deviate more than d from the prescribed time) during period x below a certain value r_{good}
	State ratings increasing	Percentage of timely ratings (i.e. the time at which the rating has been performed does not deviate more than d from the prescribed time) increasing during a certain period x
	State ratings stable	Percentage of timely ratings (i.e. the time at which the rating has been performed

		does not deviate more than d from the prescribed time) stable within certain bounds in period x
	State ratings decreasing	Percentage of timely ratings (i.e. the time at which the rating has been performed does not deviate more than d from the prescribed time) decreasing during a certain period x
Homework timely completion	Homework timely completion good	Percentage of timely complete homework assignments (i.e. the time at which the homework has been completed does not deviate more than d from the prescribed time) during period x above a certain value h_{good}
	Homework timely completion bad	Percentage of timely complete homework assignments (i.e. the time at which the homework has been completed does not deviate more than d from the prescribed time) during period x below a certain value h_{good}
	Homework timely completion increasing	Percentage of timely homework completions (i.e. the time at which the homework has been completed does not deviate more than d from the prescribed time) increasing during a certain period x
	Homework timely completion stable	Percentage of timely homework completions (i.e. the time at which the homework has been completed does not deviate more than d from the prescribed time) stable within certain bounds in period x
	Homework timely completion decreasing	Percentage of timely homework completions (i.e. the time at which the homework has been completed does not deviate more than d from the prescribed time) decreasing during a certain period x
Homework successfulness	Homework percentage filled in good	Percentage of homework measurements indicating a substantial part of homework has been finished (i.e. the percentage of homework assignment filled in is above a certain threshold h) during period x above a certain percentage value h_{good_compl}
	Homework percentage filled in bad	Percentage of homework measurements indicating a substantial part of homework has been finished (i.e. the percentage of homework assignment filled in is above a certain threshold h) during period x below a certain percentage value h_{good_compl}
	Homework percentage filled in increasing	Percentage of homework measurements indicating a substantial part of homework

		has been finished (i.e. the percentage of homework assignment filled in is above a certain threshold h) increasing during a certain period x
	Homework percentage filled in stable	Percentage of homework completed when submitting stable within certain bounds in period x
	Homework percentage filled in decreasing	Percentage of homework completed when submitting decreasing during a certain period x
Chapter timely read	Chapter timely read good	Percentage of chapter read measurements (i.e. the time at which the chapter has been completed does not deviate more than d from the prescribed time) during period x above a certain percentage value h_{good_compl}
	Chapter timely read bad	Percentage of chapter read measurements (i.e. the time at which the chapter has been completed does not deviate more than d from the prescribed time) during period x below a certain percentage value h_{good_compl}
	Chapter timely read increasing	Percentage of timely chapter completions (i.e. the intake time does not deviate more than d from the time indicated in the drug regime) increasing during a certain period x
	Chapter timely read stable	Percentage of timely chapter completions (i.e. the intake time does not deviate more than d from the time indicated in the drug regime) stable within certain bounds in period x
	Chapter timely read completion decreasing	Percentage of timely chapter completions (i.e. the intake time does not deviate more than d from the time indicated in the drug regime) decreasing during a certain period x

2.2.3 Trends: Therapy Specific Measurements

Medicine intake: For the medicine intake it is important to notice the trends in the adherence behavior of the patient. In this case, the only trend addressed is how high the percentage of adherence is for the patient with respect to a certain medicine.

Table 15. Trends for medicine intake therapy

State	Trend	
Monitored medicine adherence	Medicine adherence good for medicine m	Percentage of intake compliant with drug regime (i.e. the intake time does not

		deviate more than d from the time indicated in the drug regime) during period x above a certain percentage value med_{good}
	Medicine adherence bad for medicine m	Percentage of intake compliant with drug regime (i.e. the intake time does not deviate more than d from the time indicated in the drug regime) during period x below a certain percentage value med_{good}
	Medicine adherence increasing	Medicine adherence increasing during a certain period x
	Medicine adherence stable	Medicine adherence stable within certain bounds during a certain period x
	Medicine adherence decreasing	Medicine adherence decreasing during a certain period x

Behavioral activation. For the behavioral activation the trends with respect to the amount of activities planned are important as well as the overall percentage of the activities actually performed.

Table 16. Trends for behavioral activation therapy

State	**Trend**	
Activities scheduled	Activities scheduled good	Number of activities scheduled during a period x above value $acts_{good}$
	Activities scheduled bad	Number of activities scheduled during a period x below value $acts_{good}$
	Activities scheduled increasing	Number of activities increasing during a certain period x
	Activities scheduled stable	Number of activities stable within certain bounds during a certain period x
	Activities scheduled decreasing	Number of activities decreasing during a certain period x
Activities performed percentage	Activities percentage increasing	Percentage of activities conducted increasing during a certain period x
	Activities percentage stable	Percentage of activities conducted stable within certain bounds during a certain period x
	Activities percentage decreasing	Percentage of activities conducted decreasing during a certain period x
	Activities percentage good	Percentage of activities conducted during a period x above value act_{good}
	Activities percentage bad	Percentage of activities conducted during a period x below value act_{good}
Activity rating	Ratings for activities going up	Ratings for activities conducted increasing during a certain period x
	Ratings for activities stable	Ratings for activities conducted stable within certain bounds during a certain

		period x
	Ratings for activities going down	Rating for activities conducted decreasing during a certain period x
	Rating for activities generally good	Percentage of positive ratings for activities (i.e. the value of the rating is above a certain threshold h) during period x above a certain percentage value $actr_{good}$
	Rating for activities generally bad	Percentage of positive ratings for activities (i.e. the value of the rating is above a certain threshold h) during period x below a certain percentage value $actr_{good}$

Cognitive restructuring. For the cognitive restructuring, both the trends with respect to the registered thoughts are important, as well as the trends of the belief in the thoughts and the strength of the emotions.

Table 17. Trends for cognitive restructuring therapy

State	Trend	
Thoughts registered	Has been able to register a number of thoughts	Number of registered thoughts during period x above a certain value p_{good}
	Has not been able to register sufficient thoughts	Number of registered thoughts during period x below a certain value p_{good}
Belief in thoughts	Belief in negative thoughts high	Rating of belief in thoughts during a period x above value th_{good}
	Belief in negative thoughts low	Rating of belief in thoughts during a period x below below th_{good}
	Belief in negative thoughts increasing	Rating of belief in thoughts increasing during a certain period x
	Belief in negative thoughts stable	Rating of belief in thoughts stable within certain bounds during a certain period x
	Belief in negative thoughts decreasing	Rating of belief in thoughts decreasing during a certain period x
Emotions about thoughts	Emotions high	Rating of emotions during a period x above value emo_{good}
	Emotions low	Rating of emotions during a period x below value emo_{good}
	Emotions increasing	Rating of emotions increasing during a certain period x
	Emotions stable	Rating of emotions stable within certain bounds during a certain period x
	Emotions decreasing	Rating of emotions decreasing during a certain period x
Thoughts challenged	Has been able to register a number of challenges to thoughts	Number of registered challenges during period x above a certain value p_{good}
	Has not been able to register sufficient challenges to thoughts	Number of registered challenges during period x below a certain value p_{good}

Problem solving. Regarding the problem solving module, the crucial element is to identify problems, and to try and solve them. Hence, the therapy specific measures concern the ease with which problems are identified, and how well they are solved.

Table 18. Trends for problem solving therapy

State	Trend	
Problems identified	Has been able to define a number of problems	Number of identified problems during period x above a certain value p_{good}
	Has not been able to identify sufficient problems	Number of identified problems during period x below a certain value p_{good}
Problem solution quality	Important problems are solved in a satisfactory way	Average of problem solution ratings for important problems during period x above a certain value pq_{good}
	Important problems are not solved in a satisfactory way	Average of problem solution ratings for important problems during period x below a certain value pq_{bad}
Problem solution time	Problems are solved rapidly	Average of problem solution time during period x above a certain value pt_{good}
	Problems are solved slowly	Average of problem solution time during period x below a certain value pt_{bad}

Exercise therapy. The exercise therapy module is very close to the behavioral activation module and is shown in Table 19.

Table 19. Trends for behavioral activation therapy

State	Trend	
Exercises scheduled	Exercises scheduled good	Number of exercises scheduled during a period x above value exs_{good}
	Exercises scheduled bad	Number of exercises scheduled during a period x below value exs_{good}
	Exercises scheduled increasing	Number of exercises increasing during a certain period x
	Exercises scheduled stable	Number of exercises stable within certain bounds during a certain period x
	Exercises scheduled decreasing	Number of exercises decreasing during a certain period x
Exercises performed percentage	Exercises percentage increasing	Percentage of exercises conducted increasing during a certain period x
	Exercises percentage stable	Percentage of exercises conducted stable within certain bounds during a certain period x
	Exercises percentage decreasing	Percentage of exercises conducted decreasing during a certain period x

	Exercises percentage good	Percentage of exercises conducted during a period x above value ex_{good}
	Exercises percentage bad	Percentage of exercises conducted during a period x below value ex_{bad}
Exercise rating	Ratings for exercises going up	Ratings for exercises conducted increasing during a certain period x
	Ratings for exercises stable	Ratings for exercises conducted stable within certain bounds during a certain period x
	Ratings for exercises going down	Rating for exercises conducted decreasing during a certain period x
	Rating for exercises generally good	Percentage of positive ratings for activities (i.e. the value of the rating is above a certain threshold h) during period x above a certain percentage value exr_{good}
	Rating for exercises generally bad	Percentage of positive ratings for activities (i.e. the value of the rating is above a certain threshold h) during period x below a certain percentage value exr_{good}

Relapse prevention. For the relapse prevention there are no specific trends that are maintained, also due to the fact that the measurements are not performed on a sufficiently regular basis.

2.3. Patient State Aggregated Trends

The trends identified above say something about the development over time of one specific aspect of the patient. To draw conclusions about the situation of the patient or the therapy involvement and success in general, those aspects have to be combined, i.e. aggregated. This aggregation is done on two dimensions:

- **patient state**: the integrated assessment of the mental and physical wellbeing of the patient;
- **therapeutic involvement**: the overall assessment of the patient's involvement in the therapy.

For each of the trends identified in Section 2.2, it is specified to what extent they contribute to the aggregated patient state and the aggregated therapeutic involvement.

In the Sections 2.3.1 to 2.3.3 first the *abstracted trends* are defined, i.e., it is specified how an abstracted assessment on both dimensions can be derived for the *mental and physical patient state*, for the *generic therapeutic involvement*, and for each of the *therapy specific trends*. This is calculated using particular influences: a '+' symbol signals a positive contribution (where the number of '+' symbols indicate the strength of the influence), and a '-' symbol a negative contribution. The influence relations are based upon input obtained from experts in the mental health domain. These have based their input on knowledge of relationships between concepts as present in the literature in combination with their own experience in experiments with various types of therapy.

2.3.1 Abstracted trends: Mental and physical patient state

The following abstractions about the patient state can be derived from the trends on individual aspects of the mental and physical patient state.

Table 20. From trends regarding patient state to general patient state

Indicators	**Patient state**				
	General level during period		*General trends during period*		
	Good	Bad	Improving	Stable	Degrading
Patient generally stressed	-	+			
Patient occasionally stressed					
Patient not stressed	+	-			
Patient mood improving			+++		
Patient mood stable				+++	---
Patient mood getting worse			---		+++
Patient mood generally good	+++	---			
Patient mood generally bad	---	+++			
Patient becoming more active			++		--
Patient activity level stable				++	
Patient becoming less active			--		++
Patient generally active	--	++			
Patient generally inactive	++	--			
Patient socially becoming more active			+		-
Patients socially stable				+	
Patient socially becoming less active			-		+
Patient generally socially active	+	-			
Patient generally socially inactive	-	+			
Patients sleep quality increasing			++		--
Patients sleep quality stable				++	
Patient sleep quality decreasing			--		++
Patient generally sleeping with high quality	++	--			
Patient generally sleeping with low quality	--	++			

2.3.2 Abstracted trends: Generic Therapeutic involvement

The following abstractions about the therapeutic involvement can be derived from the trends on individual aspects of therapy progress covered in the generic therapeutic measurements.

Table 21. From trends regarding generic therapeutic elements to general therapeutic state

Indicators	**Therapeutic involvement**				
	General level during period		*General trends during period*		
	Good	Bad	Improving	Stable	Degrading
State ratings good	+++	---			
State ratings bad	---	+++			
State ratings increasing			+++		---
State ratings stable				+++	
State ratings decreasing			---		+++
Homework timely completion good	++	--			
Homework timely completion bad	++	--			
Homework timely completion increasing			++		--
Homework timely completion stable				++	
Homework timely completion decreasing			--		++
Homework percentage filled in good	++	--			
Homework percentage filled in bad	++	--			
Homework percentage filled in increasing			++		--
Homework percentage filled in stable				++	
Homework percentage filled in decreasing			--		++
Chapter timely read good	++	--			
Chapter timely read bad	++	--			
Chapter timely read increasing			++		--
Chapter timely read stable				++	
Chapter timely read completion decreasing			--		++

2.3.3 Abstracted trends: Therapy specific progress

The therapy specific trends might contribute to both the current state of the patient as well as the therapeutic state. For each of the specific therapies the influence relations are shown below.

Medicine intake. The various influences of medicine intake are shown in Table 21. For medicine, it is assumed that the trends cannot be directly linked to the general therapeutic involvement.

Table 22. From trends regarding medicine intake to general therapeutic state

Indicators	**Therapeutic involvement**				
	General level during period		*General trends during period*		
	Good	Bad	Improving	Stable	Degrading
Medicine adherence good for medicine m	++	--			
Medicine adherence bad for medicine m	--	++			
Medicine adherence increasing			++		--
Medicine adherence stable				++	
Medicine adherence decreasing			--		++

Behavioral activation. For behavioral activation, it is assumed that some trends are indicative for the therapeutic involvement, whereas others are indicative for the patient state. More in specific, the scheduling and percentage of activities performed are assumed to be related to the therapeutic involvement, whereas the actual rating of the activities (i.e. having the feeling that an activity was fun to do) and again the percentage performed are assumed to be indicators for the general patient state.

Table 23. From trends regarding behavioral activation to general therapeutic state

Indicators	**Therapeutic involvement**				
	General level during period		*General trends during period*		
	Good	Bad	Improving	Stable	Degrading
Activities scheduled good	++	--			
Activities scheduled bad	--	++			
Activities scheduled increasing			++		--
Activities scheduled stable				++	
Activities scheduled decreasing			--		++
Activities percentage good	++	--			
Activities percentage bad	--	++			
Activities percentage increasing			++		--
Activities percentage stable				++	
Activities percentage decreasing			--		++

Table 24. From trends regarding behavioral activation to general patient state

Indicators	**Patient state**

	General level during period		*General trends during period*		
	Good	Bad	Improving	Stable	Degrading
Activities percentage increasing			++		--
Activities percentage stable				++	
Activities percentage decreasing			--		++
Activities percentage good	++	--			
Activities percentage bad	--	++			
Ratings for activities going up			++		--
Ratings for activities stable				++	
Ratings for activities going down			--		++
Rating for activities generally good	++	--			
Rating for activities generally bad	--	++			

Cognitive restructuring. For cognitive restructuring, the amount of thoughts and challenges to negative thoughts that are registered is assumed to be indicative for the involvement of the patient. The belief, emotions and challenges to negative thoughts are relevant to determine the general state of the patient. Table 25 and 26 explain these dependencies.

Table 25. From trends regarding cognitive restructuring to general therapeutic state

Indicators	**Therapeutic involvement**				
	General level during period		*General trends during period*		
	Good	Bad	Improving	Stable	Degrading
Has been able to register a number of thoughts	++	--			
Has not been able to register sufficient thoughts	--	++			
Has been able to register a number of challenges to thoughts	++	--			
Has not been able to register sufficient challenges to thoughts	--	++			

Table 26. From trends cognitive restructuring solving to general patient state

Indicators	**Patient state**				
	General level during period		*General trends during period*		
	Good	Bad	Improving	Stable	Degrading
Has been able to register a number of thoughts	-	+			
Has not been able to register					

sufficient thoughts					
Belief in negative thoughts high[4]	--	++			
Belief in negative thoughts low	++	--			
Belief in negative thoughts increasing			--		++
Belief in negative thoughts stable				++	
Belief in negative thoughts decreasing			++		--
Emotions high[4]	--	++			
Emotions low	++	--			
Emotions increasing			--		++
Emotions stable				++	
Emotions decreasing			++		--
Has been able to register a number of challenges to thoughts	+	-			
Has not been able to register sufficient challenges to thoughts	-				

Problem solving. For problem solving, the identification of problems is assumed to be indicative for the involvement of the patient, but also for the general state of the patient. Furthermore, the fact that people are able to solve the problems says something about both states as well. Table 26 and 27 present an overview.

Table 26. From trends regarding problem solving to general therapeutic state

Indicators	**Therapeutic involvement**				
	General level during period		*General trends during period*		
	Good	Bad	Improving	Stable	Degrading
Has been able to define a number of problems	++	--			
Has not been able to identify sufficient problems	--	++			
The important problems are solved in a satisfactory way	++	--			
The important problems are not solved in a satisfactory way	--	++			

Table 27. From trends regarding problem solving to general patient state

Indicators	**Patient state**	
	General level during period	*General trends during period*

[4] Note that a low belief in negative thoughts and low emotions is the desired situation

	Good	Bad	Improving	Stable	Degrading
Has been able to define a number of problems	++	--			
Has not been able to identify sufficient problems	--	++			
The important problems are solved in a satisfactory way	++	--			
The important problems are not solved in a satisfactory way	--	++			

Exercise therapy. For exercise therapy the same influences are assumed as have been shown for behavioral activation. These are presented in Table 28 and 29 below.

Table 28. From trends regarding exercise therapy to general therapeutic state

Indicators	**Therapeutic involvement**				
	General level during period		*General trends during period*		
	Good	Bad	Improving	Stable	Degrading
Exercises scheduled good	++	--			
Exercises scheduled bad	--	++			
Exercises scheduled increasing			++		--
Exercises scheduled stable				++	
Exercises scheduled decreasing			--		++
Exercises percentage good	++	--			
Exercises percentage bad	--	++			
Exercises percentage increasing			++		--
Exercises percentage stable				++	
Exercises percentage decreasing			--		++

Table 29. From trends regarding exercise therapy to general patient state

Indicators	**Patient state**				
	General level during period		*General trends during period*		
	Good	Bad	Improving	Stable	Degrading
Exercises percentage increasing			++		--
Exercises percentage stable				++	
Exercises percentage decreasing			--		++
Exercises percentage good	++	--			
Exercises percentage bad	--	++			
Ratings for exercises going up			++		--
Ratings for exercises stable				++	
Ratings for exercises going down			--		++
Rating for exercises generally good	++	--			

Rating for exercises generally bad	--	++			

The abstracted trends above can be combined via an averaging mechanism to derive a combined abstracted trend on both the *patient state* and the *therapeutic involvement*.

2.3.4 Aggregated trend: characterization of the situation

The combined abstracted trends about the *patient state* and the *therapeutic involvement* together can then be used to give an overall evaluation of the situation, namely how well the patient is doing, and how the therapy is of influence on this wellbeing of the patient.

To give a structured overview of all possible situations, a table is created with all possible patient states on the horizontal axis, and all possible levels of therapeutic involvement on the vertical axis. This results in the following 16 situations:

Table 30. Aggregated characterizations of the situation

		Patient state					
		Good			*Bad*		
Therapeutic involvement		*Increasing*	*Stable*	*Decreasing*	*Increasing*	*Stable*	*Decreasing*
Good	*Increasing*	Successful therapy	Patient no longer sees improvement and wants to improve more, therefore tries to improve intensity of therapy	Patient starts feeling worse, and tries to be more involved in therapy, which does not help	Successful therapy	Patient does not see improvements , and tries to be more involves in therapy, which does not help	Patient does not see improvements, and tries to be more involves in therapy, which does not help
	Stable	Successful therapy	Successful therapy	Unsuccessful therapy	Successful therapy	Unsuccessful therapy	Unsuccessful therapy
	Decreasing	Patient feeling well for some time no, and no longer sees the necessity of therapy	Patient feeling well for some time no, and no longer sees the necessity of therapy	Patient felt better for a while, and was no longer motivated. As a consequence the mood is not going down	Patient starts feeling better, but does not attribute this to the therapy	Patient does not feel better, and therefore becomes less involved in the therapy	Patient does not feel better, and therefore becomes less involved in the therapy
Bad	*Increasing*	Patient feels fine, and now has the feeling therapy might help	Patient is no feeling good, and now ready to be more involved in the therapy	Patient starts to feel worse and becomes more involved in the therapy	Patient is getting more involved in the therapy and feels better	Patient feels bad, and tries to improve involvement in the therapy to feel better	Patient feels bad, and tries to improve involvement in the therapy to feel better
	Stable	The patient is feeling better and better, but not due to the therapy	Patient feeling ok, but is not very involved in the therapy	Patient starts feeling worse, but does not have the feeling that therapy might help	Despite the low involvement, the patient mood is improving	Unsuccessful therapy	Patient does not like the therapy, and is feeling worse and worse

	Decreasing	The patient is feeling a lot better and even improving, but does not contribute it to the therapy	Patient stable at a sufficient level, therefore decides to become even less involved in therapy	Patient starts feeling worse and does not have any hope that the therapy can help, therefore becomes even less involved	The patient starts to feel better, but not due to the therapy	The patient is not feeling better, and thinks the therapy is not helping at all	Patient gets less involved in the therapy, and is feeling worse and worse

3. Abstracting ontological elements: A Formal Approach

In order to relate the various parts in the ontology and investigate the trends over time, an expressive temporal logic is needed. The demands for dynamic modeling and analysis approaches suitable for this particular problem is nontrivial. In particular, the possibility of modeling of a system at different aggregation levels in both discrete and continuous ways is needed in order to determine such trends. Furthermore, numerical expressivity is required for modeling systems such as the one addressed in this document, as several sensors provide us with quantitative values. Moreover, for specifying qualitative aspects of a system, the modeling language should be able to express logical relationships between parts of a system. The choice has been made to use the language called TTL (for Temporal Trace Language), which has recently been developed at the VU University cf. [41].

3.1. Desiderata

Desiderata for analysis techniques include both the generation and formalization of simulated and empirical trajectories or traces, as well as analysis of complex dynamic properties of such traces and relationships between such properties. A *trace* as used here represents a temporally ordered sequence of states of the agent (in this case the patient following therapy). Each state is characterized by a number of *state properties*, for example a qualitative logical property or a state variable having a certain numerical value. The data hereby comes from the sensor devices as used by the patient. This notion of a trace contrasts to that of the Mazurkiewicz traces known in Theoretical Computer Science [32] used to analyze the behavior of Petri Nets. Mazurkiewicz traces represent restricted partial orders over algebraic structures with a trace equivalence relation.

The desiderata for modeling languages and analysis techniques described above are not easy to fulfill. On the one hand, high expressivity is desired, on the other hand computationally feasible analysis techniques are demanded. Providing automated analysis support limits the expressivity of the modeling language. For example, the expressivity may be limited to difference and differential equations as in DST (excluding logical relationships), or to propositional modal temporal logics (excluding numerical relationships). In the former case, calculus can be exploited to do simulation and analysis based on continuous variables only [35]. In the latter case, simulation is based on a specific logical executable format, which does not allow expressions involving continuous variables (e.g., executable temporal logic [2]). Other proposals use a number of dedicated formal languages with limited expressiveness and related to them analysis techniques for checking different particular static and dynamic aspects of a system. For example, the KORSO [14] methodology for the development of correct software uses this approach for checking the structural consistency of a model and dynamic aspects of execution. The languages used in the KORSO project describe different formats of system specifications, relations between them (e.g., by refinement based on proof obligations) and the temporal development of these specifications for all phases of the software life cycle. However, to guarantee the overall correctness of a system requires properties to be expressed in more than one language with different types of semantics. Thus, the problem of verification across different not related proof systems arises that is not addressed in the KORSO project.

The problem of checking relationships between dynamic properties of a system (e.g. the trends in certain metrics in the ICT4Depression system), is essentially the problem of justifying entailment relations between sets of properties defined at different aggregation levels of a system's representation. In general, entailment relations can be established either by logical proof procedures or by checking properties of a higher aggregation level on the set of all theoretically possible traces generated by executing a system specification that consists of properties of a lower aggregation level (i.e., by performing model checking [17, 31, 38]). To make checking relationships between dynamic properties feasible, expressivity of the language for these properties has to be limited. However, checking properties on a given set of traces of practical size (instead of all theoretically possible ones), obtained empirically or by simulation, is computationally much cheaper. In the scenario of this project, empirical traces are in fact the only source available for the analysis of the trends of the patient. Therefore, in that case the language for these properties can be more expressive, as shown in this paper for the sorted predicate logic temporal trace language TTL. TTL fulfils all of the identified desiderata and can be used both for formalization of empirical traces of the patient and for analysis of properties on traces.

3.2. A Language for Dynamic Properties of Agents and Multi-Agent Systems

The Temporal Trace Language (TTL) and its software environment as developed by the VU University Amsterdam have been designed to fulfill the desiderata and compromises as discussed in Section 3.1. TTL is meant as a tool to formally specify and analyze properties of models for agents and multi-agent systems at different aggregation levels, varying from the smallest executable (computation) steps within an agent model to overall global properties of an agent or multi-agent system. Possibilities for analysis include specification of inter-level relations between dynamic properties at different aggregation levels, checking properties against given (empirical or simulation) traces of the model, and integration of qualitative and quantitative aspects. In this case, the language is used to analyze trends in empirical traces of depressed patients.

The assumption that the dynamics of an agent or multi-agent system can be described as an evolution of states of the agents and their environment over time, served as point of departure in the development of TTL. This assumption also underlies modal temporal logics, see e.g., [2, 17, 20, 31, 38]. TTL shares some similarities with situation calculus [36] and event calculus [27]. A more detailed comparison of TTL to other well-known formalisms for modeling system dynamics is presented in Section 3.xx. Time in TTL is assumed to be linearly ordered. In the case of the ICT4Depression project, it is assumed that the states in this case concern particular elements that have been obtained via the sensor devices. Depending on the application, time may be dense (e.g., the real numbers), or discrete (e.g., the set of integers or natural numbers or a finite initial segment of the natural numbers), or any other form with a linear ordering. An agent interacts with a dynamic environment via its *input* and *output* (interface) states. At its input the agent receives observations from the environment whereas at its output it generates actions that can change a state of the environment.

3.2.1 Agent states and state properties

An agent state at a certain point in time as used here is an indication of which of the state properties of the agent and its environment (e.g., observations and actions) are true (hold) at that time point. For specifying state properties for the input, output, internal, and external states of an agent A, state ontologies, named InOnt(A), OutOnt(A), IntOnt(A), and ExtWorldOnt respectively, are used which are specified by a number of sorts, sorted constants, variables, functions and predicates (i.e., a signature in order-sorted predicate logic; e.g., [33, 36]).
State properties are formulae constructed using a standard multi-sorted first-order predicate language based on such ontologies. For example, a state property expressed as a predicate mood may belong to IntOnt(A), whereas the atom has_gps_location(52.34, 4.87) may belong to ExtWorldOnt.

3.2.2 Sorts and atoms for dynamic properties

To characterize the dynamics of the agent and the environment, *dynamic properties* relate properties of states at certain points in time. To enable reasoning about the dynamic properties of arbitrary systems the language TTL includes special sorts, such as:

TIME	a set of linearly ordered time points
STATE	a set of all state names of an agent system

TRACE a set of all trace names; a trace or a trajectory can be thought of as a timeline with a state for each time point
STATPROP a set of all state property names
PART a set of all names for "parts" of agents (e.g., inputs, outputs, internals) and the world, to which state properties are related.

Throughout the paper, variables such as t, t1, t2, t', t" stand for variables of the sort TIME; and variables such as γ, $\gamma1$, $\gamma2$ stand for variables of the sort TRACE. A state of an agent is related to a state property via the satisfaction relation

$\models$: STATE x STATPROP

formally defined as a binary infix predicate (or by holds as a binary prefix predicate in the software environment). For example,

"in the output state of agent A in trace γ at time t property p holds"

is formalized by

state(γ , t, output(A)) $\models$ p.

Here function symbols are used such as:

state:	TRACE x TIME x PART $\rightarrow$ STATE
output:	AGENT $\rightarrow$ PART
input:	AGENT $\rightarrow$ PART
internal:	AGENT $\rightarrow$ PART

If the indication of an agent aspect is not essential, the third argument is left out: state(γ, t) $\models$ p, thus using a function

state: TRACE x TIME $\rightarrow$ STATE

Both state(γ, t, output(A)) and p are terms of the TTL language. TTL terms are constructed by induction in a standard way for sorted predicate logic from variables, constants and function symbols typed with TTL sorts.

3.2.3 Dynamics properties

Dynamic properties are expressed by TTL-formulae inductively defined by:

(1) If v_1 is a term of sort STATE, and u_1 is a term of the sort STATPROP, then holds(v1, u1) is an atomic TTL formula.
(2) If $\tau1$, $\tau2$ are terms of any TTL sort, then $\tau1 = \tau2$ is an atomic TTL formula.
(3) If t1, t2 are terms of sort TIME, then t1 < t2 is an atomic TTL formula.

(4) The set of well-formed TTL-formulae is defined inductively in a standard way based on atomic TTL-formulae using Boolean propositional connectives and quantifiers.

For example, the dynamic property

'in any trace γ, if at any point in time t1 agent A observes that it is dark in the room, whereas earlier a light was on in this room, then there exists a point in time t2 after t1 such that at t2 in the trace γ agent A switches on a lamp'

is expressed in formalized form as:

$$\forall t1\ [\ [\ state(\gamma, t1, input(A)) \models observed(dark\_in\_room)\ \&$$
$$\exists t0<t1\ [\ state(\gamma, t0, input(A)) \models observed(light\_on)]$$
$$\Rightarrow\ \exists t2 \geq t1\ state(\gamma, t2, output(A)) \models performing\_action(switch\_on\_light)\ ]$$

Within TTL the following abbreviation is used for summation:

$$\Sigma k{:}S\ \ case(\varphi, v1, v2) = v$$

Here for any formula φ, the expression $case(\varphi, v1, v2)$ indicates the value v1 if φ is true, and v2 otherwise. The formula as mentioned is an abbreviation for a formula involving conjunctions over subsets {k1,… , kn} of sort S of known size N:

$$\wedge_{n=1,\dots N}\ \ \wedge_{i=1,\dots,n}\ \varphi(k_i)\ \wedge\ \forall k{:}S\ [\ [\ \wedge_{i=1,\dots,n}\ k \neq k_i\ ] \Rightarrow \neg\ \varphi(k)\ ] \Rightarrow\ v = n*v1 + (N-n)*v2$$

This abbreviation is very useful in practice. Within the software environment special facilities have been implemented to evaluate such statements.

As TTL uses order-sorted predicate logic as a point of departure, it inherits the standard semantics of this variant of predicate logic. That is, the semantics of TTL is defined in a standard way, by interpretation of sorts, constants, functions and predicates, and a variable assignment. In addition the semantics involves some specialized aspects. As a number of standard sorts are present, the elements of these sorts are limited to instances of specified terms in these sorts, as is usual, for example, in logic programming semantics. For example, for the sort TIME it is assumed that in its semantics its elements consist of the time points of the fixed time frame chosen. Moreover, for the sort TRACE, it is assumed that in its semantics its elements consist of a (limited) number of traces named by constants. Furthermore, for the sort STATPROP for state properties it is assumed that in its semantics its elements consist of the set of terms denoting the propositions built in a chosen state language (this is called reification). A full description of the technical details of TTL's semantics is beyond the scope of the current paper. For this purpose, see [37].

By executing dynamic properties traces can be generated and visualized, for example as in Figure 2. Here, the time frame is depicted on the horizontal axis. The names of predicates are shown on the vertical axis. A dark box on top of the line indicates that the predicate is true during that time period.

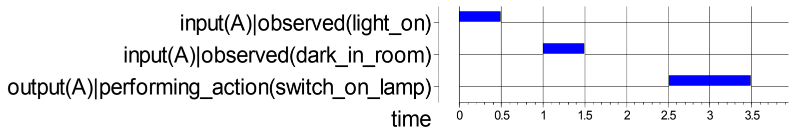

Figure 2. Example visualization of a trace

3.3. Modeling and Analysis of Hybrid Systems in TTL

Hybrid systems incorporate both continuous and discrete components. The dynamics of the continuous components can be described by differential equations, those of discrete components can be represented by finite-state automata. The continuous and discrete dynamics influence each other. In particular, the input to the continuous dynamics is the result of some function of the discrete state of a system; whereas the input of the discrete dynamics is determined by the value of the continuous state.

A modeling method for hybrid systems should be capable of expressing both quantitative and qualitative properties of the system and integrating them into one model. TTL satisfies this requirement. Qualitative aspects of systems can be directly expressed by logical TTL properties, which essentially describe temporal relations between system states occurring over time. Quantitative aspects represented by systems of differential equations can be expressed in TTL using discrete or dense time frames in the following manner. As an example, Euler's method, see [34], for solving differential equations is modeled in TTL. Euler's method approximates a differential equation $dy/dt = f(y)$ with the initial condition $y(t_0)=y_0$ by a difference equation $y_{i+1}=y_i+h*f(y_i)$ ($i \geq 0$ is the step number and $h>0$ is the integration step size). This equation can be modeled in TTL in the following way:

$$\forall\gamma\ \forall t\ \forall v\text{: VALUE } state(\gamma , t) \models has\_value(y, v) \quad \Rightarrow \quad state(\gamma , t+1) \models has\_value(y, v + h \bullet f(v))$$

States properties specify the respective values of y at different time points and the difference equation is modeled by a transition rule from the current to the successive state. The traces γ satisfying the above dynamic property are the solutions of the difference equation. More precise and stable numerical approximation methods (e.g., Runge-Kutta, dynamic step size, see [34]) can be expressed in TTL in a similar manner.

3.4. Analysis of Trace Conditioning in TTL

The example considered in this section illustrates how TTL can be used for the analysis of continuous models of complex systems. This example is taken from [7]. In that paper, TTL is used to analyze the temporal dynamics of trace conditioning. In general, research into conditioning is aimed at revealing the principles that govern associative learning. An important issue in conditioning processes is the adaptive timing of the conditioned response to the appearance of the unconditioned stimulus. This feature is most apparent in an experimental procedure called *trace conditioning*. In this procedure, a trial starts with the presentation of a *warning stimulus* (S1, comparable to a conditioned stimulus). After a blank interval, called the *foreperiod*, an *imperative stimulus* (S2, comparable to an unconditioned stimulus) is presented to which the participant responds as fast as possible. The *reaction time* to S2 is used as an estimate of the conditioned state of preparation at the moment S2 is presented. In this case, the conditioned response obtains its maximal strength, here called *peak level*, at a moment in time, called *peak time*, that closely corresponds to the moment the unconditioned stimulus occurs.

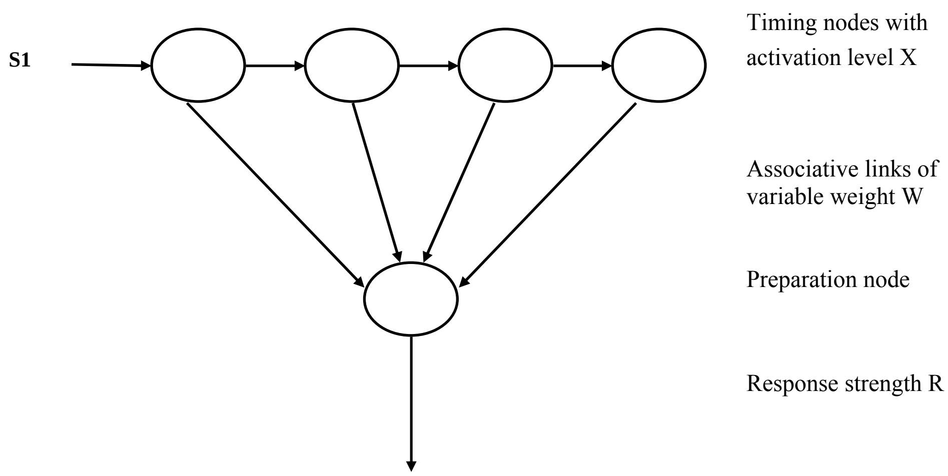

Figure 3. Structure of Machado's conditioning model

Machado [30] developed a basic model that describes the dynamics of these conditioning processes in terms of differential equations. The structure of this model is shown in Figure 3. The model posits a layer of *timing nodes* and a single *preparation node*. Each timing node is connected both to the next (and previous) timing node and to the preparation node. The connection between each timing node and the preparation node (called *associative link*) has an adjustable weight associated to it. Upon the presentation of a warning stimulus, a cascade of activation propagates through the timing nodes according to a regular pattern. Owing to this regularity, the timing nodes can be likened to an internal clock or pacemaker. At any moment, each timing node contributes to the activation of the preparation node in accordance with its activation and its corresponding weight. The activation of the preparation node reflects the participant's preparatory state, and is as such related to reaction time. The weights reflect the state of conditioning, and are adjusted by learning rules, of which the main principles are as follows. First, *during* the foreperiod extinction takes place, which involves the decrease of weights in real time in proportion to the activation of their corresponding timing nodes. Second, *after* the presentation of the imperative stimulus a process of reinforcement takes over, which involves an increase of the weights in accordance with the current activation of their timing nodes, to preserve the importance of the imperative moment. Machado describes the more detailed dynamics of the process by a mathematical model (based on linear differential equations), representing the (local) temporal relationships between the variables involved. For example,

$$dX(t,n)/dt = \lambda X(t,n-1) - \lambda X(t,n)$$

expresses how the activation level of the n-th timing node X(t+dt,n) at time point t+dt relates to this level X(t,n) at time point t and the activation level X(t,n-1) of the (n-1)-th timing node at time point t. Similarly, as another example,

$$dW(t,n)/dt = -\alpha X(t,n)W(t,n)$$

relates the n-th weight W(t+dt,n) at time point t+dt to this weight W(t,n) at time point t and the activation level X(t,n) of the n-th timing node at time point t.

In [7], a number of dynamic properties relevant for trace conditioning have been formalized in TTL. These properties were taken from the existing literature on conditioning, such as [29], in which they were mainly expressed informally. TTL turned out useful to express these properties in a formal manner. An example of such a property (taken from [29], p.372) is given below, both in informal, semi-formal and in formal notation:

Global Hill Preparation

Informal: 'The state of conditioning implicates an increase and decay of response-related activation as a critical moment is bypassed in time'.

Semi-formal: 'In trace γ, if at t1 a stimulus s1 starts, then the preparation level will increase from t1 until t2 and decrease from t2 until t1 + u, under the assumption that no stimulus occurs too soon (within u time) after t1.' Formal:

```
has_global_hill_prep(γ:TRACE, t1,t2:TIME, u:INTEGER) ≡
∀t',t":TIME ∀p',p":REAL
[ state(γ, t1) |= stimulus_occurs  & ¬ stimulus_starts_within(γ, t1, t1+u)  &
state(γ, t') |= preparation_level(p')  & state(γ, t") |= preparation_level(p")
⇒  [t1 ≤ t' < t" ≤ t2  &  t" ≤ t1 + u  ⇒   p' < p" ]  &
    [t2 ≤ t' < t" ≤ t1 + u              ⇒   p' > p" ] ]
```

Here, stimulus_starts_within is defined as follows:

stimulus_starts_within(γ:TRACE, t1,t2:TIME) ≡
    ∃t:TIME [state(γ, t) |== stimulus_occurs & t1 < t < t2]

These (and various similar) properties were automatically verified using the TTL checker tool against a number of (empirical and simulation) traces. Among these properties were properties that compare different traces, such as:

'the conditioned response takes more time to build up and decay and its corresponding asymptotic value is lower when its corresponding critical moment is more remote from the warning signal.' (cf. [29])

Such properties cannot be expressed, for example, in modal temporal logics, just like familiar properties such as 'exercise improves skill', expressing that the more intensive a training history, e.g., of an athlete, the better the skill will be.

3.5. Software Environment

This section presents the software environment[5] developed in SWI-Prolog to support the process of specification and automated verification of dynamic properties on a limited set of traces. The software environment consists of two closely integrated tools: the Property Editor and the Checker Tool.

The Property Editor provides a user-friendly way of building and editing properties in TTL. By means of graphical manipulation and filling in forms a TTL specification can be constructed. TTL specifications may also be provided as plain text. When a TTL specification is created, the Checker Tool can be used to verify automatically whether a TTL property holds for a given set of traces. User interaction with the tools involves three separate actions:

1. Loading, editing, and saving a TTL specification in the Property Editor (see Figure 4).
2. Loading and inspecting traces to be checked by activating the Trace Manager.
3. Checking a property against a set of loaded traces by the Checker Tool. The property is compiled and checked, and the result is presented to the user. If a property is not satisfied by a set of traces, then a counter-example is provided to the user, which identifies the cause of failure.

Note that the traces that are loaded in step 2 can be either traces produced by simulations (see [8]) or empirical traces. Empirical traces may be obtained by formalizing empirical data from log-files produced by information systems or from results of experiments. Within the area of *Requirements Engineering* methods have been described to aid the

[5] The software can be downloaded from the following URL: http://www.cs.vu.nl/~wai/TTL.

modeler in formalization of scenarios (which can be considered as informal traces) and requirements. For example, in [21] it is described how requirements and scenarios initially formulated informally in natural language and/or graphical elements can be restructured into a structured natural language format, which then can be reformulated more easily in a formal language. In the structured language format the keywords of TTL are made explicit, e.g., keywords for state properties, the input and output references and the temporal succession relations. In this way not only complex behavioral properties can be formulated, but also scenarios that help in modeling new complex systems. A structured semi-formal representation of a scenario is obtained by the following steps:

- explicitly distinguish *input and output* concepts in the scenario description,
- define (domain) *ontologies* for the input and output information,
- represent the temporal structure described implicitly in a sequence of events.

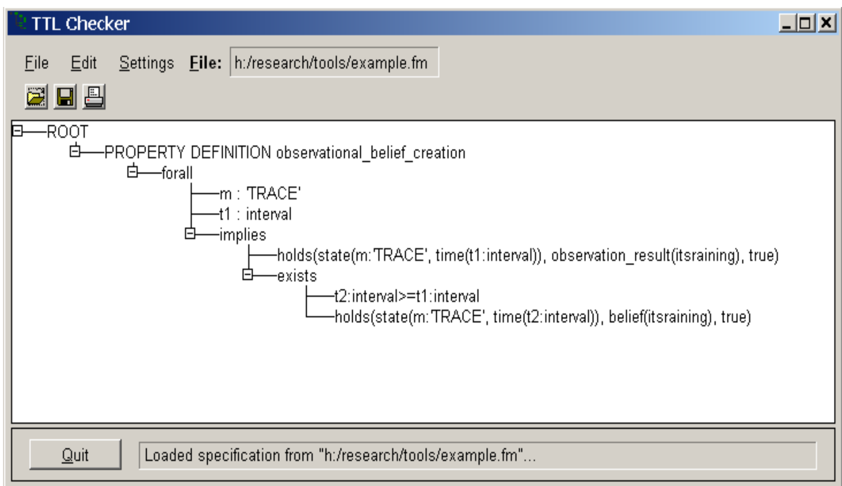

Figure 4. The TTL Checking Environment

A simple example in informal representation is the following:

The temperature and pressure are high.
A red alert is generated and the heater is turned off.

This can be reformulated into a more structured form as follows.

- input: temperature is high, pressure is high
- output: red alert, situation is explosive
- input: situation to be resolved
- output: heater is turned off

Note that in the example an implicit intermediate state property (the meaning of the red alert) is made explicit. A formalization can be made by using formal ontologies for the concepts used, and by formalizing the relationships. More precisely, formalization of a scenario on the basis of a structured semi-formal representation is achieved by:

- choosing *formal ontologies* for the input and output information
- formalization of the *temporal structure*

This results in a formal temporal trace γ for the scenario.

state(γ, 1, input(S))	|=	high(temperature)
state(γ, 1, input(S))	|=	high(pressure)
state(γ, 2, output(S))	|=	red_alert
state(γ, 3, input(S))	|=	explosive_situation_to_be_resolved
state(γ, 4, output(S))	|=	turn_off(heater)

For a more extensive discussion about the transition from informal to formal, see [21].

The following sections provide more details on the implementation of the software environment. In particular, Section 3.5.1 describes the implementation of the TTL Editor and Section 3.5.2 discusses the verification procedure underlying the TTL checker.

3.5.1 Format of a TTL specification in the TTL Property Editor

A TTL specification constructed in the TTL Property Editor consists of a number of user-defined property definitions and sort definitions. A property definition consists of a header (property name and arguments, i.e., prop_name(v1:s1, v2:s2)) and a body (a TTL formula). Arbitrary sorts may be defined by enumerating their elements.

A TTL formula is constructed from atomic TTL formulae by conjunction, (Formula1 and Formula2), disjunction (Formula1 or Formula2), negation (not Formula), implication and quantification (forall ([v1:s1, v2:s2], Formula), exists ([v1:s1, v2:s2 < term2], Formula)).

Atomic TTL formulae correspond to user-defined properties, holds atoms (e.g., holds(state(trace1, t, output(ew)), a1 ∧ a2) or state(trace1, t, output(ew)) |= a1 ∧ a2), mathematical expressions (e.g., term1 = term2, term1 > term2) and built-in properties (i.e., complex properties encoded into the implementation language).

All TTL formulae are constructed from terms that are implemented as Prolog terms (e.g., fn(t1,t2) , n1, t1 + t3, 1.3). Constants, variables and functions from which terms are constructed should be typed with appropriate sorts. For example, each variable should be declared as variable_name: sort. The software supports a number of built-in sorts, among which sorts for integer, real and range of integers (i.e., sorts integer, real, between(i1:integer,i2:integer)), the sort for the set of all states (STATE) and the sort for the set of all traces (TRACE). Furthermore, libraries with predefined general purpose and domain-specific sorts and functions are available for creating terms.

3.5.2 Verification by the TTL Checker

After a TTL property is specified in the Editor and traces have been loaded by the Trace Manager, the Checker Tool can be used to determine if the considered property holds in the loaded traces. To perform such verification an algorithm has been developed.

The verification algorithm is a backtracking algorithm that systematically considers all possible instantiations of variables in the TTL formula under verification. However, not for all quantified variables in the formula the same backtracking procedure is used. Backtracking over variables occurring in holds atoms is replaced by backtracking over values occurring in the corresponding holds atoms in traces under consideration. Since there are a finite number of such state atoms in the traces, iterating over them often will be more efficient than iterating over the whole range of the variables occurring in the holds atoms. Formulae that contain variables quantified over infinite sorts not occurring in a holds atom cannot be checked by the TTL Checker.

As time plays an important role in TTL-formulae, special attention is given to continuous and discrete time range variables. Because of the finite variability property of TTL traces (i.e., only a finite number of state changes occur between any two time points), it is possible to partition the time range into a minimum set of intervals within which all atoms occurring in the property are constant in all traces. Quantification over continuous or discrete time variables is replaced by quantification over this finite set of time intervals.

In order to increase the efficiency of verification, the TTL formula that needs to be checked is compiled into a Prolog clause. Compilation is obtained by mapping conjunctions, disjunctions and negations of TTL formulae to their Prolog equivalents, and by transforming universal quantification into existential quantification. Thereafter, if this Prolog clause succeeds, the corresponding TTL formula holds with respect to all traces under consideration.

The complexity of the algorithm has an upper bound in the order of the product of the sizes of the ranges of all quantified variables. However, if a variable occurs in a holds atom, the contribution of that variable is no longer its range size, but the number of times that the holds atom pattern occurs (with different instantiations) in trace(s) under consideration. The contribution of an isolated time variable is the number of time intervals into which the traces under consideration are divided.

The specific optimizations discussed above make it possible to check realistic dynamic properties with reasonable performance. In particular, checking a relatively complex property involving eight different time points (see [41] for details on the formula itself) against a single trace with three state atoms occurring in the verified formula and 28 changes of atom values over time takes 0.76 sec. on a regular PC. With the increase of the number of traces with similar complexity as the first one, the verification time grows linearly: for 3 traces - 3.9 sec., for 5 traces - 6.59 sec. However, the verification time is polynomial in the number of isolated time range variables occurring in the formula under verification.

3.6. Discussion on TTL

The use of the temporal trace language TTL has a number of practical advantages. In the first place, it offers a well-defined language to formulate relevant dynamic relations in practical domains, with standard first order logic semantics. It has a high expressive power. For example, the possibility of explicit reference to *time points* and *time durations* enables modeling of the dynamics of continuous real-time phenomena, such as sensory and neural activity patterns in relation to mental properties (cf. [35]). Also difference and differential equations can be expressed. These features go beyond the expressive power available in standard linear or branching time temporal logics.

Furthermore, the possibility to quantify over traces allows for specification of *more complex adaptive behaviors*. Once can for instance think of groups of patients. As within most temporal logics, reactiveness and pro-activeness properties are specified. In addition, our language allows the specification of different types of adaptive behavior. For example, a property such as

‘exercise improves skill’

is a relative property in the sense that it involves the comparison of two alternatives for the history. Another property of this type is trust monotonicity:

‘For any two traces γ1 and γ2, if

initially in trace γ2 A’s trust is at least as high as A’s trust at t in trace γ1, and

at each time point t agent A’s experience with public transportation in γ2 at t is at least as good as A’s experience with public transportation in γ1 at t, then in trace γ2 at each point in time t, A’s trust is at least as high as A’s trust at t in trace γ1’.

∀γ1, γ2

[∀w1, w2:VALUE [state(γ1, 0) |== has_value(trust, w1) &

state(γ2, 0) ⊨ has_value(trust, w2)] ⇒ w1≤ w2] &

[∀t, ∀v1,v2:VALUE [state(γ1, t) ⊨ has_value(experience, v1) &

state(γ2, t) ⊨ has_value(experience, v2)] ⇒ v1≤ v2]]] ⇒

[∀t, ∀w1, w2:VALUE [state(γ1, t) ⊨ has_value(trust, w1) &

state(γ2, t) ⊨ has_value(trust, w2)] ⇒ w1≤ w2]]]]]

Thus, different alternative histories can be represented and compared in TTL, which is not possible in standard forms of temporal logic. Similarly, the kind of relative or comparative properties put forward in [23], such as 'the more south on the northern hemisphere, the higher the trees', as properties lacking an explanation in terms of a cause and its effects, can be expressed since our language allows comparison of different traces and different (local) restrictions within traces.

The possibility to define restrictions to *local languages for parts* of a system or the world is an important feature. For example, the distinction between internal, external and input and output languages is crucial, and is supported by the language TTL. Thereby TTL enables to quantify over system parts and the specification of system modification over time. This possibility allows to consider traces in which 'brain, body and world' are modeled in an integrative manner, and to focus on one of these aspects in the context of the overall trace [15, 16].

Finally, since state properties are used as first class citizens in the temporal trace language, it is possible to explicitly refer to them, and to quantify over them, enabling the specification of what are sometimes called *second-order properties*, which are used in part of the philosophical literature (e.g., [26]) to express functional roles related to mental properties or states.
TTL has some similarities with the situation calculus [36] and the event calculus [27], which are two well-known formalisms for representing and reasoning about temporal domains. However, a number of important syntactic and semantic distinctions exist between TTL and both calculi. In particular, the central notion of the situation calculus - a situation - has different semantics than the notion of a state in TTL. That is, by a situation is understood a history or a finite sequence of actions, whereas a state in TTL is associated with the assignment of truth values to all state properties (a "snapshot" of the world). Moreover, in contrast to the situation calculus, where transitions between situations are described by actions, in TTL actions are in fact properties of states.

Moreover, although a time line has been recently introduced to the situation calculus [36], still only a single path (a temporal line) in the tree of situations can be explicitly encoded in the formulae. In contrast, TTL provides more expressivity by allowing explicit references to different temporally ordered sequences of states (traces) in dynamic properties (e.g., the trust monotonicity property). Examples of properties in which different histories are compared are given in Section 5.

In contrast to the event calculus, TTL does not employ the mechanism of events that initiate and terminate fluents. Events in TTL are considered to be functions of the external world that can change states of components, according to specified properties of a system. Furthermore, similarly to the situation calculus, in the event calculus also only one time line is considered.

TTL can also be related to temporal languages that are often used for verification (e.g., propositional temporal logic (PTL) and linear-time logic (LTL) [3, 17, 20]). Propositional modal temporal logic can be seen as an extension of classical propositional logic by temporal operators, for a linear discrete time frame (e.g., '○', meaning "at the next moment in time", '□' meaning "at every future moment", '◊' meaning "at some future moment"). The PTL formulae can be translated into TTL formulae by replacing temporal operators by temporal relations on states. For example,

$X \rightarrow \circ Y$ is translated to $\forall\gamma, t\ \text{state}(\gamma, t) \models X \rightarrow \text{state}(\gamma, t+1) \models Y$
$X \rightarrow \Box Y$ is translated to $\forall\gamma, t\ \text{state}(\gamma, t) \models X \rightarrow \forall t1\ t1>t\ \text{state}(\gamma, t1) \models Y$
$X \rightarrow \Diamond Y$ is translated to $\forall\gamma, t\ \text{state}(\gamma, t) \models X \rightarrow \exists t1\ t1>t\ \text{state}(\gamma, t1) \models Y$

However, due to the limitations related to the quantitative expressivity of PTL, not every TTL formula can be represented as a PTL formula. In particular, this holds for TTL formulae with numerical time and arithmetic expressions.

The general idea of translation of a LTL formula into a TTL expression is rather straightforward: by replacing the temporal operators of LTL by quantifiers over time. Consider the following LTL formula

G(observation_result(itsraining) → ***F***(belief(itsraining)))

where the temporal operator ***G*** means 'for all later time points', and ***F*** 'for some later time point.' This formula is translated into the following TTL expression:

$$\forall t1\ [\ \text{state}(\gamma, t1) \models \text{observation\_result(itsraining)} \Rightarrow \exists t2 > t1\ \text{state}(\gamma, t2) \models \text{belief(itsraining)}\ ]$$

Note that the translation is not bi-directional, i.e., it is not always possible to translate TTL expressions into LTL expressions. An example of a TTL expression that cannot be translated into LTL is again the property of trust monotonicity.

Furthermore, TTL has the expressivity provided by different extensions of PTL. In particular, the extended temporal logic (ETL) [40] provides a possibility to express any property definable by a regular expression on sequences of states, which cannot be expressed in PTL. Due to the fact that the syntax of TTL provides quantifiers, predicates, and arithmetic functions, such properties can be also expressed in TTL. For example, the property "a given proposition p has to be true in every even state of a sequence" can be expressed in TTL as follows: $\forall t\ \text{state}(\gamma, 2 \bullet t) \models p$.

Furthermore, to specify and reason about qualitative properties of a system, qualitative reasoning techniques can be used [4]. The main idea of these approaches is to represent quantitative knowledge in terms of abstract, qualitative concepts. In comparison to TTL, purely qualitative languages are less expressive with respect to temporal and quantitative aspects. In [28] interesting work is presented that addresses in depth how accurately approximate quantitative models using qualitative models. However, still the issue of hybrid modeling that includes both numerical, quantitative aspects (e.g., modeled by differential and difference equations) and qualitative aspects (modeled using logics) is not considered by qualitative reasoning techniques.

To support the formal specification and analysis of dynamic properties in TTL, special software tools (the Property Editor and the Checker Tool) have been developed. The Property Editor has an intuitive graphical interface for building and editing TTL properties, and the Checker Tool employs an efficient algorithm for the formal verification of properties against a limited set of traces. Although this form of checking is not as exhaustive as model checking (which essentially means checking properties on the set of all traces generated by model execution), in return, it allows more expressivity in specifying properties.

The TTL environment has been tested and proved its value in a number of projects within different domains; e.g., [7, 9, 10, 11, 12, 13]). During this work, the TTL environment has been further developed to provide automated support.

4. Formalization of the ontology and relations for the patient and therapy

In Section 2, a lot of informal measurements were shown as well as informal specification of trends and abstraction of these measurements. This section provides a formalization of the elements introduced in Section 2, based upon the formalization approach which has been described in Section 3. First the formal terms used to represent the measurements are shown, followed by the formalization of the temporal abstraction. Finally, the aggregation to an accurate representation of the state of the patient as well as the state of the therapy is addressed in Section 4.3.

4.1. Formal Terms

In order to make a formalization of the properties possible, formal terms which represent each of the measurements should be defined. In this section, these formal forms are shown. First, the sorts used to make the formalization possible are shown in Table 31. Note that in this case, merely a sort representing the heart rate is present, and no sort for the other physiological measurements. This is due to the fact that these measurements are only used after pre-processing (e.g. recognizing a certain sensor recognizable activity).

Table 31. Sorts used to formalize the measurements, abstractions, and trends

Sort	Explanation
RATING	A number between 1 and 10
STATE_ELEMENT	An information element which concerns the mental or physical state of the patient
INTEGER	An integer
REAL	A real number
MEDICINE_TYPE	An identifier of a type of medicine
RATING_TYPE	A rating type of the rating that should be inserted by the patient, this includes: stress, mood, and sleep_quality. The other ratings are not explicitly scheduled at particular time points.
THERAPEUTIC_MEASUREMENT_ELEMENT	An element part of the therapeutic measurements
HOMEWORK_ID	An identifier of a certain piece of homework part of a therapy
CHAPTER_ID	An identifier of a particular chapter which is part of a therapy
QUESTIONNAIRE_ID	An identifier of a particular questionnaire within the system
THERAPY_TYPE	The type of therapy (e.g. problem solving, cognitive restructuring)
THERAPEUTIC_ACTIVITY	A therapeutic activity, see the activities listed in Table 6
SENSOR_RECOGNIZABLE_ACTIVITY	An activity which can be recognized by the sensor devices, see Table 7
EXERCISE	A certain exercise being performed by the patient. The SENSOR_RECOGNIZABLE_ACTIVITY is a subsort of EXERCISE
LOCATION	A location which can be recognized by the mobile phone (see Table 8)
TIME	A certain time point (e.g. in the format dd:mm:yyyy:hh:mm)
THOUGHT	A description of a thought, which can simply consist of a string of words
CHALLENGE	A description of a challenge to a negative thought, which can simply consist of a string of words
PROBLEM	A description of a problem, which can simply consist of a string of words
SEVERITY	The severity is either important or unimportant
HEART_RATE	A number between 0 and 255

The terms that are formed to represent the measurements are shown in Table 32 (mental or physical state of the patient) and Table 33 (therapeutic measurements). Note that in most of the terms time is not made explicit as this is part of the language TTL (as described in Section 3) in which statements can be made about the expression as shown below. For instance, a certain measurement of the mood level being a “4” at a time point 1 (using the term in the first row of Table 32) can be expressed as follows:

state(patient1, 1) |= mood_level(4)

Table 32. Terms describing the patient mental and physical state

Mental or physical state	**Term**	**Brief explanation**
Stress level	stress_level: RATING → STATE_ELEMENT	The measured stress level of the patient
Mood level	mood_level: RATING → STATE_ELEMENT	The self-reported mood level of the patient
Activity level	activity_level: RATING → STATE_ELEMENT	The measured activity level of the patient
Social interaction	social_interaction_level: RATING → STATE_ELEMENT	The measured social interaction level of the patient
Sleep quality	sleep_quality: RATING → STATE_ELEMENT	The self-reported sleep quality of the patient
Rating current therapy (how much does the patient still like the current therapy)	current_therapy_rating: RATING → STATE_ELEMENT	The self-reported rating of the therapy of the patient
Anxiety level	anxiety_level: RATING → STATE_ELEMENT	The self-reported rating of the anxiety level of the patient
Positivity of thoughts	positivity_of_thoughts: RATING → STATE_ELEMENT	The self-reported rating of the positivity of the thoughts of the patient
Motivation	motivation_level: RATING → STATE_ELEMENT	The self-reported rating for the motivation of the patient
Self-efficacy	self_efficacy: RATING → STATE_ELEMENT	The self-reported rating for the self efficacy of the patient
Number of GP visits	number_of_GP_visits: INTEGER → STATE_ELEMENT	The number of GP visits of the patient
Health expenses	health_expenses: REAL → STATE_ELEMENT	The health expenses the patient has had
Number of working hours	number_of_working_hours: REAL → STATE_ELEMENT	The number of hours the patient has been working
Medicine usage	medicine_usage: MEDICINE_TYPE x INTEGER → STATE_ELEMENT	The type and amount of medicine per week the

		patient is using

Table 33. Terms describing the therapeutic measurements

Therapeutic measurement	Term	Brief explanation
State rating scheduled	state_rating_scheduled: RATING_TYPE → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain rating of the patient state has been scheduled
State rating performed	state_rating_performed: RATING_TYPE → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain rating of the patient state has been performed
Homework deadline	homework_deadline: HOMEWORK_ID → THERAPEUTIC_MEASUREMENT_ELEMENT	A deadline for a certain piece of homework has been scheduled
Homework submission: percentage done	homework_submission_percentage: HOMEWORK_ID x PERCENTAGE → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain piece of homework has been submitted by the patient, of which a particular percentage has been performed
Homework submission: time spent	homework_submission_time_spent: CHAPTER_ID x INTEGER → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain piece of homework has been submitted by the patient, on which a certain amount of time (in minutes) has been spent
Chapter deadline	chapter_deadline: CHAPTER_ID → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain deadline for finishing a particular chapter has been scheduled
Chapter finished: percentage read	chapter_submission_percentage: CHAPTER_ID x PERCENTAGE → THERAPEUTIC_MEASUREMENT_ELEMENT	The patient has indicated to have finished a particular chapter, of which a certain percentage was actually displayed on the screen
Chapter finished: time spent	chapter_submission_time_spent: CHAPTER_ID x INTEGER → THERAPEUTIC_MEASUREMENT_ELEMENT	The patient has indicated to have finished a particular chapter, on which a certain amount of time (in minutes) has been spent
Questionnaire deadline	chapter_deadline: QUESTIONNAIRE_ID → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain deadline for finishing a particular questionnaire has been scheduled
Questionnaire finished: percentage answered	chapter_submission_percentage: QUESTIONNAIRE_ID x PERCENTAGE → THERAPEUTIC_MEASUREMENT_ELEMENT	The patient has indicated to have finished a particular questionnaire,

		of which a certain percentage of the questions was actually filled in
Therapies followed in the past	therapy_followed_in_past: THERAPY_TYPE → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain type of therapy has been followed by a patient in the past
Patient's preference for performing activities	general_preference_activities: RATING → THERAPEUTIC_MEASUREMENT_ELEMENT	The patient gives a particular rating for performing activities in general
Medicine intake scheduled	medicine_intake_scheduled: MEDICINE_TYPE → THERAPEUTIC_MEASUREMENT_ELEMENT	A medicine intake for the specified type of medicine has been scheduled
Medicine intake performed	medicine_intake_performed: MEDICINE_TYPE → THERAPEUTIC_MEASUREMENT_ELEMENT	A medicine of the specified type has been taken by the patient
Activity scheduled	activity_scheduled: THERAPEUTIC_ACTIVITY x TIME x TIME → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain therapeutic activity has been scheduled with a certain start and end time. Time has been added in these terms to distinguish the same activities scheduled at different time points
Activity performed (automated)	activity_performed_automated: SENSOR_RECOGNIZABLE_ACTIVITY → THERAPEUTIC_MEASUREMENT_ELEMENT	The sensors have recognized a certain sensor recognizable activity between a start and end time
Activity performed (manual)	activity_performed_manual: THERAPEUTIC_ACTIVITY → THERAPEUTIC_MEASUREMENT_ELEMENT	The patient manually inserted that a certain activity has been performed (based upon a question posed by the system)
Activity rating scheduled	activity_rating_scheduled: THERAPEUTIC_ACTIVITY x TIME x TIME → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain rating of the therapeutic activity which should have been performed between the indicated start and end time has been scheduled
Activity rated	activity_rating_performed: THERAPEUTIC_ACTIVITY x TIME x TIME → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain rating of the therapeutic activity that has been scheduled between the indicated start and end time has been performed

Location event	patient_location: LOCATION → THERAPEUTIC_MEASUREMENT_ELEMENT	The patient resides at a certain location
Thought identification event	thought_identification: THOUGH → THERAPEUTIC_MEASUREMENT_ELEMENT	The patient has registered a (negative) thought
Belief in thought rated	belief_rating_performed: THOUGHT x TIME → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain rating of the belief in a though has been performed at a certain timepoint
Emotion about thought rated	emotion_rating_performed: THOUGHT x TIME → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain rating of the emotions about a though has been performed at a certain timepoint
Thought challenge event	challenge_identification: CHALLENGE → THERAPEUTIC_MEASUREMENT_ELEMENT	The patient has registered a challenge to a negative thought
Problem identified	problem_identified: PROBLEM x SEVERITY → THERAPEUTIC_MEASUREMENT_ELEMENT	A problem has been identified, and this problem is of a certain severity
Problem solved	problem_solved: PROBLEM x RATING → THERAPEUTIC_MEASUREMENT_ELEMENT	A problem has been solved with a certain satisfaction level
Exercise scheduled	exercise_scheduled: EXERCISE x TIME x TIME → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain exercise has been scheduled with a certain start and end time
Exercise performed (automated)	exercise_performed_automated: SENSOR_RECOGNIZABLE_ACTIVITY → THERAPEUTIC_MEASUREMENT_ELEMENT	The sensors have recognized a certain exercise (which is recognizable by the sensors) between a start and end time
Exercise performed (manual)	exercise_performed_manual: EXERCISE → THERAPEUTIC_MEASUREMENT_ELEMENT	The patient manually inserted that a certain exercise has been performed (based upon a question posed by the system)
Exercise rating scheduled	exercise_rating_scheduled: EXERCISE x TIME x TIME → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain rating of the exercise which should have been performed between the indicated start and end time has been scheduled
Exercise rated	exercise_rating_performed: THERAPEUTIC_ACTIVITY x TIME x TIME → THERAPEUTIC_MEASUREMENT_ELEMENT	A certain rating of the therapeutic activity that has been scheduled between the indicated start and end time has been performed

Heart rate	heart_rate: HEART_RATE → THERAPEUTIC_MEASUREMENT_ELEMENT	The patient has a certain heart rate
Problems	See row *problem identified*	
General activity level	general_activity_level: RATING → THERAPEUTIC_MEASUREMENT_ELEMENT	The patient indicates the general activity level

4.2. Temporal Abstraction

Of course, just representing single measurements about the patient state does not give a good picture of the overall functioning of the human over a longer period. Therefore, a form of temporal abstraction is applied. Before this temporal abstraction can be applied however, certain combinations of statements need to be made in order to be able to make such temporal expressions. More in specific:

- Sensor recognizable activities in combination with location information should be mapped to therapeutic activities in order to allow them to be mapped.
- Planning of particular tasks should be mapped to actual performance of the tasks (e.g. chapter reading, homework, activities being conducted, etcetera).

Both these aspects are explained first, followed by the generic specification of the trends as they have been identified in Section 2.2.

4.2.1 Mapping activities

As has been expressed in Section 2, patients have a list of activities they can select from in the behavioral activation therapy (Table 6). These activities are however expressed on a fairly high level, for instance "Go out to a restaurant". Of course, such an activity cannot easily be recognized by the sensors part of the ICT4Depression system. Activities that are generally recognizable by these sensors for instance include "walking", but also a location can be recognized (e.g. "at a restaurant"). In order to be able to map these different levels of activity recognition, and hence, to judge whether the patient is actually performing the activities without having to explicitly ask for it, a formal approach is presented below. Note that the assumption is made that only for a small subset of the therapeutic activities a mapping can be created due to the nature of the activities (e.g. think of "learning something new"). For the activities for which no mapping can be found, a question will simply be posed to the patient.

The first step in creating the mapping is to know what sensor recognizable activities and locations are indicators for a certain activity, and how they relate in a temporal fashion (e.g. going out to a restaurant means going to the restaurant first, staying there for a while, and going back again). Therefore, the following sorts and terms are introduced:

SENSORY_INDICATION = LOCATION ∪ SENSOR_RECOGNIZABLE_ACTIVITY
required_for: SENSORY_INDICATION x DURATION x DURATION x {1, 2, ..., n} x THERAPEUTIC_ACTIVITY

The idea of these sorts is as follows: The sort SENSORY_INDICATION represents the various sensory indicators that there are (in this case a combination of the locations, and the sensory recognizable activities). Furthermore, in the term required_for it is indicated what sensory indication should be measured in relationship with a certain therapeutic activity, and what duration it should have (a duration between a certain minimum and a certain maximum duration). Also, it is indicated in what order these indicators should be measured, expressed by the number on the fourth argument (which is a limited set of integers, depending on the maximum length of such a sequence). To make things more concrete, let's return to our previous example. The therapeutic activity *go out to a restaurant* can be decomposed into the following combination of terms (assuming that the person always walks to a restaurant, and that the closest restaurant is 5 minutes away):

required_for(walking, 5, 30, 1, go_out_to_a_restaurant)
required_for(location_restaurant, 60, 180, 2, go_out_to_a_restaurant)
required_for(walking, 5, 30, 3, go_out_to_a_restaurant)

This expresses that the patient should perform the activity walk between 5 and 30 minutes first, followed by a location indication that the patient is in a restaurant for at least 60 minutes, and at most 180 minutes. Thereafter, the patient should be walking back again. Sometimes, you might also consider the location restaurant being sufficient, multiple definitions for these requirements can exist. In order to formalize the recognition of the therapeutic activity based upon the statements above, formalizations in TTL have been specified on a generic level. First, the definition of a general indicator is expressed:

indicator(γ:TRACE, t:TIME, L:LOCATION) ≡
state(γ, t) |= patient_location(L)

indicator(γ:TRACE, t:TIME, S:SENSOR_RECOGNIZABLE_ACTIVITY) ≡
state(γ, t) |= activity_performed_automated(S)

This expresses that a location indicator holds within a trace γ at time point t in case a measurement is present in the system that the patient was at this location at that time point. For the sensor recognizable activity the same holds. Now that the definition of an indicator is present, it can be determined whether a certain therapeutic activity has been performed. The definition for this is shown below.

therapeutic_activity_during_interval(γ:TRACE, T:THERAPEUTIC_ACTIVITY, t_{start}:TIME, t_{end}:TIME) ≡
∀S1:SENSORY_INDICATION, D_{min1}:DURATION, D_{max1}:DURATION, I1:{1, 2, ..., n}
[required_for(S, D_{min1}, D_{max1}, I1, T) ⇒
∃t:TIME [indicator_measured_correctly(γ, t, t_{start}, t_{end}, S, D_{min1}, D_{max1}, T) &
∀S2:SENSORY_INDICATION, D_{min2}:DURATION, D_{max2}:DURATION, I2:INTEGER
[I2 < I1 & I2 > 0 ⇒

$\neg\exists t_{prime} > t$ [indicator_measured_correctly(γ, t_{prime}, t_{start}, t_{end}, S, D_{min2}, D_{max2}, T)]] &
[I2 > I1 $\Rightarrow$
$\neg\exists t_{prime} < t$ [indicator_measured_correctly(γ, t_{prime}, t_{start}, t_{end}, S, D_{min2}, D_{max2}, T)]
]
]
]

The property is expressed as follows: given a certain start and end time, a therapeutic activity to be recognized, and a trace of the patient, a therapeutic activity is detected during a certain period in case for each required sensory indication a time point can be found such that this indicator is measured correctly. Furthermore, for each sensory indication before the current indication (i.e. an integer smaller than the sequence number I1 accompanying the current sensory recognizable activity) there should not be a time point after the current activity at which it is measured correctly. The same holds for sensory indicators that should follow after the current indication: these should not occur before the currently considered activity.

each required sensory indicator that is required is indeed measured correctly, and these activities are in the appropriate order. The measurement of an activity indicator being correct is defined below.

indicator_measured_correctly(γ:TRACE, t:TIME, t_{start}:TIME, t_{end}:TIME, S:SENSORY_INDICATION, D_{min}:DURATION, D_{max}:DUARION, T:THERAPEUTIC_ACTIVITY) ≡
$\exists D_{actual}$:DURATION
$t \geq t_{start}$ & $t + D_{actual} \leq t_{end}$ & $D_{actual} \geq D_{min}$ & $D_{actual} \leq D_{max}$ &
$\forall t_{prime} \geq t$ & $t_{prime} \leq t + D_{actual}$ [indicator(γ, t_{prime}, S)]

An indicator is measured according to the definition if a duration can be found such that the duration in combination with the time point at which the element is supposed to hold stays within the boundaries of the duration and interval under investigation, and the indicator is measured during the entire duration.

4.2.2 Matching schedules

As has been expressed before, in many of the therapies, certain schedules are present for which trends are identified that indicate whether the patient complies with the schedule or not. In this section, the general outline of how the schedules are connected with the actual behavior of the human, is given.

Ratings. Ratings are frequently scheduled, and in case the patient performs the rating, this information is also stored within the ICT4Depression system. Essentially, for the state ratings, the following terms are essential to measure the compliance of the patient to the rating schedule:

state_rating_scheduled: RATING_TYPE
state_rating_performed: RATING_TYPE

The idea is that the patient complies in case it performs the rating within a certain margin from the schedule. Assuming that the maximum deviation can be *d*, the rating is satisfactory in case:

compliant_rating(γ:TRACE, t:TIME, R:RATING_TYPE, d:DURATION) ≡
state(γ, t) |= state_rating_scheduled(R) ⇒
∃t_{prime}:TIME < t + d & t_{prime} > t - d [state(γ, t_{prime}) |= state_rating_performed(R)]

In other words, in case a rating of the particular type takes place at a time point within the interval *[t-d, t+d]* where *t* is the scheduled time, the rating is considered to comply to the schedule.

Homework. For the Homework, exactly the same principle can be applied to identify whether the homework has been performed appropriately:

compliant_homework(γ:TRACE, t:TIME, d:DURATION) ≡
∀H:HOMEWORK_ID
[state(γ, t) |= homework_deadline(H) ⇒
∃P:PERCENTAGE, t_{prime}:TIME < t + d & t_{prime} > t-d
[state(γ, t_{prime}) |= homework_submission_percentage(H, P)]]

Note that this property will only be satisfied in case all homework assignments scheduled for that specific time point are performed. The current assumption is however that no more than one homework assignment is due at a certain time point.

Chapters. For the Chapters, the same holds:

compliant_chapter(γ:TRACE, t:TIME, d:DURATION) ≡
∀C:CHAPTER_ID
[state(γ, t) |= chapter_deadline(C) ⇒
∃P:PERCENTAGE, t_{prime}:TIME < t + d & t_{prime} > t-d
[state(γ, t_{prime}) |= chapter_submission_percentage(C, P)]]

Questionnaire. And the same principle applies for the questionnaire:

compliant_questionnaire(γ:TRACE, t:TIME, d:DURATION) ≡
∀Q:QUESTIONNAIRE_ID
[state(γ, t) |= questionnaire_deadline(Q) ⇒
∃P:PERCENTAGE, t_{prime}:TIME < t + d & t_{prime} > t-d

[state(γ, t_{prime}) |= questionnaire_submission_percentage(Q, P)]]

Medicine intake. For medicine intake, the setting of the allowed derivation will heavily depend upon the type of medicine. The following property expresses the compliance, given a certain duration *d* assumed:

compliant_medicine(γ:TRACE, t:TIME, d:DURATION) ≡
∀M:MEDICINE_ID
[state(γ, t) |= medicine_intake_scheduled(M) ⇒
∃t_{prime}:TIME < t + d & t_{prime} > t-d
[state(γ, t_{prime}) |= medicine_intake_performed(M)]]

Activities. Essentially, with respect to the activity scheduling and rating, two combinations need to be made, namely the rating of the activities (which will not be shown, it follows the same line of reasoning as shown above) and the matching of the performance of the activities. The mapping of activities is performed as follows:

compliant_activity_automated(γ:TRACE, t:TIME, d:DURATION) ≡
∀A:THERAPEUTIC_ACTIVITY, t_1, t_2:TIME
[state(γ, t_1) |= activity_scheduled(A, t_1, t_2) ⇒
∃t_{1prime}:TIME > t_1 - d & t_{2prime} ≤ t_2 + d
therapeutic_activity_during_interval(γ, t, t_{1prime}, t_{2prime})]

The definition of the last statement has been provided in Section 4.2.1. In case the automated approach does not work, it can also be checked in a manual form:

compliant_activity_manual(γ:TRACE, t:TIME, d:DURATION) ≡
∀A:THERAPEUTIC_ACTIVITY, t_1, t_2:TIME
[state(γ, t_1) |= activity_scheduled(A, t_1, t_2) ⇒
∃t_{1prime}:TIME > t_1 - d & t_{2prime} ≤ t_2 + d
[∀$t_{between}$:TIME ≥ t_{1prime} & $t_{between}$ ≤ t_{2prime} [state(γ, $t_{between}$) |= activity_performed_manual(A)]]]

Exercises. For exercises the properties are expressed in the same way as indicated above, except that the automatically detected activities do not need to be recognized in a complex fashion. Below, the two definitions are given:

compliant_exercise_automated(γ:TRACE, t:TIME, d:DURATION) ≡
∀A:SENSOR_RECOGNIZABLE_ACTIVTY, t_1, t_2:TIME
[state(γ, t_1) |= exercise_scheduled(A, t_1, t_2) ⇒
∃t_{1prime}:TIME > t_1 - d & t_{2prime} ≤ t_2 + d
[∀$t_{between}$:TIME ≥ t_{1prime} & $t_{between}$ ≤ t_{2prime}
[state(γ, $t_{between}$) |= exercise_performed_automated(A)]]]

compliant_exercise_manual(γ:TRACE, t:TIME, d:DURATION) ≡
∀A:SENSOR_RECOGNIZABLE_ACTIVTY, t_1, t_2:TIME
[state(γ, t_1) |= exercise_scheduled(A, t_1, t_2) ⇒
∃t_{1prime}:TIME > t_1 - d & t_{2prime} ≤ t_2 + d
[∀$t_{between}$:TIME ≥ t_{1prime} & $t_{between}$ ≤ t_{2prime}
[state(γ, $t_{between}$) |= exercise_performed_manual(A)]]]

For the ratings the formalization is again not shown, but these completely comply with the expression shown before.

4.2.3 Trends

Informally, the trends that are distinguished (also given the intermediate steps that have been explained above) have been discussed in Section 2. In this Section, a full formalization of the properties that describe the trends will be shown. It briefly concerns the following properties:

- Increasing during a period x.
- Decreasing during a period x.
- Stable (fluctuations within certain boundaries) during a period x.
- Average over a period x is above a threshold th.
- Average over a period x is below a threshold th.
- Percentage of cases above a threshold th during period x.
- Percentage of cases below a threshold th during period x.

Note that the formalization of these properties will be shown on a generic level (using the sort MEASUREMENT representing the measurements that can be performed, including the combinations expressed in 4.2.2) as the application of these generic concepts to the domain of ICT4Depression is quite straightforward. Furthermore, this also allows the reuse of these generic expressions for trends in different domains.

Increasing during a period x. In order to express that an increasing trend can be seen with respect to some measurements in the ICT4Depression system is not a trivial matter. Certain outliers might occur in the data that need to be filtered out, and when looking at individual measurements such outliers can be quite difficult to detect.

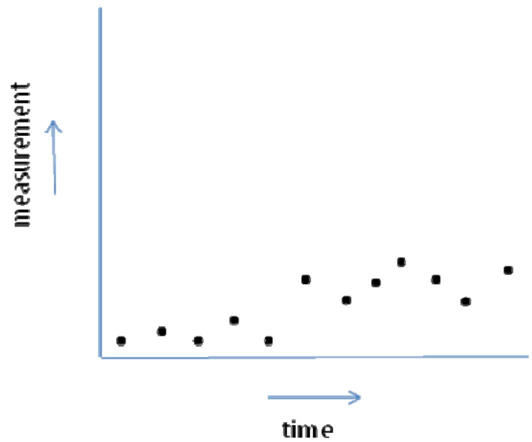

Figure 5. Example increasing trend

For instance, when looking at Figure 5, a clear increasing trend can be seen, but also many outliers that prohibit a strict property with respect to an increasing measurement from being satisfied. Of course, many different techniques can be applied to detect the increasing trends, e.g. the fitting of a linear curve through the data making use of the method of least squares [42]. Another approach is to divide the overall period under investigation into smaller intervals, calculate the average, and see whether these averages are monotonically increasing. The latter approach is much closer to the logical approach used throughout this deliverable. However, formalizations of both approaches will be shown, followed by a brief discussion on the pros and cons of each of the approaches.

Linear Approximation using least squares error

The idea behind a linear approximation using the least squares error is that the data can be described by means of a linear function:

$y = a \cdot x + b$

Different values for the parameters *a* and *b* can be tried, and the value for these parameters that minimizes the least squares error between the data points and the function is selected. First, a property expressing the error in a single time point is expressed, namely the squared difference between the actual rating of the measurement and the predicted value of the measurement (in case the time point is within the specified interval).

single_squared_error(a:REAL, b:REAL, t_{start}:TIME, t_{end}:TIME, t:TIME, γ:TRACE, M:MEASURMENT, s_s_error:REAL) ≡

∀R:REAL

[[$t \geq t_{start}$ & $t \leq t_{end}$ & state(γ, t) |= has_value(M, R)] ⇒ s_s_error = $(R - (a * t + b))^2$]

The overall squared error is then defined as follows:

total_squared_error(a:REAL, b:REAL, t_{start}:interval, t_{end}:interval, γ:TRACE, M:MEASUREMENT, s_error:REAL) ≡
s_error = $\sum_{\forall t:TIME,\ R:REAL}$ case(single_squared_error(γ, t_{start}, t_{end}, t, a, b, R), R, 0)

In the aforementioned formalization, the case(p, a, b) operator evaluated to a in case property p holds, or b in case the property does not hold. Since the idea is to minimize the squared error, the following property expresses which combination of parameters *a* and *b* have the lowest squared error:

lowest_squared_error(a:REAL, b:REAL, t_{start}:interval, t_{end}:interval, γ:TRACE, M:MEASUREMENT) ≡
∀tse:REAL
[total_squared_error(a, b, t_{start}, t_{end}, γ, M, tse) ⇒
∀a':LIMITED_REAL ≠ a, b':_LIMITED_REAL ≠ b, tse':REAL
[total_squared_error(a, b, t_{start}, t_{end}, γ, M, tse') ⇒ tse' ≥ tse]]

Note that in the property above, the sort LIMITED_REAL is used to illustrate the problem that this logical formula requires that each instance of the sort real (i.e. infinitely many) must be passed. The LMITED_REAL sort could contain a subset of all reals to make this property feasible. Finally, the trend can be said to be increasing in case the value for parameter *a* with the lowest squared error value has a value greater than 0 (i.e. it is an increasing linear function).

increasing_trend(γ:TRACE, t_{start}:TIME, t_{end}:TIME, M:MEASUREMENT) ≡
∀a, b:LIMITED_REAL
[lowest_squared_error(a, b, t_{start}, t_{end}, γ, M) ⇒ a > 0]

Division in small intervals

Given the more logical approach presented in this paper, the least squared error method as expressed in the properties above suffers from computational problems due to the fact that all instances (at least to a certain depth) of potential real values need to be passed. Therefore, an alternative approach is taken. The main principle used in that approach is to divide the measurements in certain (relatively small) intervals, calculate averages of the measurements during these intervals (whereby outliers are not considered), and look at the trend of these averages over multiple of these intervals. For this trend, a strict definition of increasing (e.g. monotonically increasing) can be used.

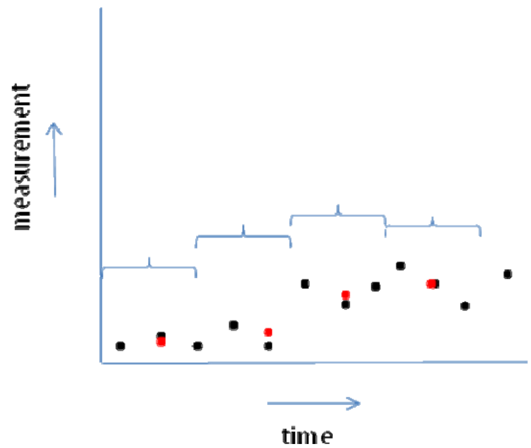

Figure 6. Example steps trend identification

Figure 6 illustrates how the measurements are abstracted. A certain interval (indicated by the bracket) is taken and the average value is calculated (expressed by means of the red dot). Thereafter, it is verified whether these averages increase over time. In order to formalize the identification of the increasing trend, the first step is to calculate the averages during the small intervals:

average_value(γ:TRACE, t_{start}:TIME, t_{end}:TIME, avg:REAL, M:MEASUREMENT) $\equiv$
$\exists$uncorrected_avg, number_of_elements, corrected_number_of_elements, sd:REAL
number_of_elements = $\sum_{\forall t:TIME,\ R:REAL}$ case(current_value(γ, t_{start}, t_{end}, M, t, R), 1, 0) &
uncorrected_avg = ($\sum_{\forall t:TIME,\ R:REAL}$ case(current_value(γ, t_{start}, t_{end}, M, R), t, R, 0)) /
number_of_elements &
sd = $\sqrt{}$ (($\sum_{\forall t:TIME,\ R:REAL}$ case(current_value(γ, t_{start}, t_{end}, M, t, R), (R – uncorrected_avg)^2, 0)) /
number_of_elements) &
corrected_number_of_elements =
$\sum_{\forall t:TIME,\ R:REAL}$ case(non_deviant_value(γ, t_{start}, t_{end}, M, avg, sd, t, R), 1, 0)
avg = ($\sum_{\forall t:TIME,\ R:REAL}$ case(non_deviant_value(γ, t_{start}, t_{end}, M, avg, sd, t, R), R, 0)) /
corrected_number_of_elements

It can be seen that the calculation of the average value during the interval is done in a number of steps. First, the number of measurements within the considered interval is calculated, and the average of all these elements is considered. In order to do so, the following formula determines whether and what number should be added to the sum:

current_value(γ:TRACE, t_{start}:TIME, t_{end}:TIME, M:MEASUREMENT, t:TIME, R:REAL) $\equiv$
$t \geq t_{start}$ & $t \leq t_{end}$ & state(γ, t) |= has_value(M, R)

Note that the has_value(M,R) can be replaced with a term representing the specific measurement (as introduced in Section 4.1). The standard deviation of the data is also calculated. After the uncorrected average as well as the standard deviation has been calculated, the outliers can be removed. The rationale behind the removal of the outliers is that the aim is to identify the general trend, and not focus on incidental negative ratings. To enable removal of the outliers, a formula is expressed which indicates whether a certain measurement is acceptable or not (i.e. whether it is an outlier or not). This formula is shown below:

non_deviant_value(γ:TRACE, t_{start}:TIME, t_{end}:TIME, avg:REAL, sd:REAL, M:MEASUREMENT, R:REAL) ≡
$t \geq t_{start}$ & $t \leq t_{end}$ & state(γ, t) |= has_value(M, R) & R ≤ (avg + 2 * sd) & R ≥ (avg - 2 * sd)

In this case, a measurement is assumed to be too deviant (i.e. an outlier) in case it deviates more than 2 times the standard deviation. Of course, more advanced approaches can easily be incorporated within the formula.

Given that the corrected average can now be calculated, the formula for increasing can be expressed:

increasing_trend(γ:TRACE, t_{start}:TIME, t_{end}:TIME, D:DURATION, M:MEASUREMENT) ≡
∀I:INTEGER < ((t_{start} – t_{end})/D), R1, R2:REAL
[average_value(γ, (t_{start} + I*D), (t_{start} + (I + 1)*D), R1, M) &
average_value(γ, (t_{start} + (I + 1)*D), (t_{start} + (I + 2)*D), R2, M)] ⇒
R2 > R1

This expresses that a trend is considered increasing when for all intervals of duration D that are present in the expressed overall interval the value of the next interval of duration D is higher than the previous one.

Comparison between the two approaches
Due to the logical nature of the approach presented in this deliverable the approach to approximate the parameters of a linear function is very difficult. The central issue is that all instances of the sort used to approximate the linear function (i.e. the values for *a* and *b*) need to be passed, which is very inefficient. Therefore, a subset of these values needs to be selected, however this significantly reduces the quality of the approximation. Ideally, one would want to start with a relatively coarse grained set of values, find the region of the parameters which describes the data best, and then work on a more fine-grained scale. In this logical approach this is however not feasible. The second approach with division into smaller intervals and calculation of corrected averages within these smaller intervals does give results in the same direction, but is much closer to the nature of the logical approach used throughout this deliverable.

Decreasing during a period x. For the decreasing trend, the same approach can be followed which has been explained for the increasing trend. In this case, the approach to divide the period into smaller intervals (i.e. the second approach just introduced) has been chosen. The only formalization that needs to be altered compared to that definition is the definition of the trend itself:

decreasing_trend(γ:TRACE, t_{start}:TIME, t_{end}:TIME, D:DURATION, M:MEASUREMENT) ≡
∀I:INTEGER < ((t_{start} – t_{end})/D), R1, R2:REAL
[average_value(γ, (t_{start} + I*D), (t_{start} + (I + 1)*D), R1, M) &
average_value(γ, (t_{start} + (I + 1)*D), (t_{start} + (I + 2)*D), R2, M)] ⇒
R2 < R1

Stable trend. For the identification of the stable trend, it is assumed that a certain percentage of deviation is allowed. Once a measurement is outside of these bounds, the stable trend property is not satisfied.

stable_trend(γ:TRACE, t_{start}:TIME, t_{end}:TIME, deviation:PERCENTAGE, M:MEASUREMENT) ≡
∃avg, number_of_elements:REAL
[number_of_elements = $\sum_{\forall t:TIME,\ R:REAL}$ case(current_value(γ, t_{start}, t_{end}, M, t, R), 1, 0) &
avg = ($\sum_{\forall t:TIME,\ R:REAL}$ case(current_value(γ, t_{start}, t_{end}, M, R), t, R, 0)) / number_of_elements &
∀t:TIME, R:REAL
[current_value(γ, t_{start}, t_{end}, M, R) ⇒
[R ≥ (1+(deviation/100) * avg) & R ≤ (1-(deviation/100) * avg)

The property simply specifies a calculation of the average, and all measurements should be within the range of the average plus the allowed deviation above and below this average.

Average over a period x is above a threshold th. The calculation of the average value being above a certain threshold *th* is based upon the previously identified approach to calculate the average value during a certain period. Hereby, again, the same approach of calculating the average (with removal of the outliers) is used:

average_above_threshold(γ:TRACE, t_{start}:TIME, t_{end}:TIME, M:MEASUREMENT, th:THRESHOLD) ≡
∀R:REAL
[average_value(γ, t_{start}, t_{end}, R, M) ⇒ R > th]

Average over a period x is below a threshold th. This property is formalized as follows:

average_below_threshold(γ:TRACE, t_{start}:TIME, t_{end}:TIME, M:MEASUREMENT, th:THRESHOLD) ≡
∀R:REAL
[average_value(γ, t_{start}, t_{end}, R, M) ⇒ R < th]

Percentage of cases above a threshold th during period x. For the derivation of the percentage of cases above a threshold h, the outliers will no longer be removed as the idea behind the property is to identify what percentage of the total measurements are indeed above this threshold.

percentage_above_threshold(γ:TRACE, t_{start}:TIME, t_{end}:TIME, th:threshold, perc:REAL, M:MEASUREMENT) ≡
∃number_of_elements:REAL
number_of_elements = $\sum_{\forall t:TIME,\ R:REAL}$ case(current_value(γ, t_{start}, t_{end}, M, t, R), 1, 0) &
perc = ($\sum_{\forall t:TIME,\ R:REAL}$ case([current_value(γ, t_{start}, t_{end}, M, R) & R > th], t, 1, 0)) / number_of_elements

The definition of the property essentially expresses the number of measurements that have been performed during the specified interval, and thereafter determines how many of these measurements are above the specified threshold.

Percentage of cases below a threshold th during period x. This property is similar to the expression above, except that now the values below the threshold are summed:

percentage_below_threshold(γ:TRACE, t_{start}:TIME, t_{end}:TIME, th:threshold, perc:REAL, M:MEASUREMENT) ≡
∃number_of_elements:REAL
number_of_elements = $\sum_{\forall t:TIME,\ R:REAL}$ case(current_value(γ, t_{start}, t_{end}, M, t, R), 1, 0) &
perc = ($\sum_{\forall t:TIME,\ R:REAL}$ case([current_value(γ, t_{start}, t_{end}, M, R) & R < th], t, 1, 0)) / number_of_elements

Once the trends are identified, the next step is to combine all the trends into a coherent picture of how the patient is currently doing.

4.3. Aggregation/combination

In Section 2.3 the contributions of the various trends to the overall picture of the functioning of the human and the therapeutic involvement have been expressed by means of positive and negative influence relations. The trends themselves have been formalized in Section 4.2, and in this section the formalization of the contributions of these trends to the overall judgment of the current state of the patient will be shown. In order to describe the patient's state, several additional sorts are introduced to describe the influence of the trends as shown in Table 34.

Table 34. Sorts used to describe aggregates of the patient and therapy

Sort	Explanation
MEASUREMENT	The measurements that can be performed within the ICT4Depression

	system, including the measurements that involve the matching of schedules with the patient behavior (as expressed in Section 4.1.2)
TREND	Identifier of the trends (i.e. all trends expressed formally in Section 4.2)
AGGREGATE_VALUE	The value of the aggregate considered, this can have the value increasing, stable, decreasing, good, and bad.
INFLUENCE	The influence of a trend upon the aggregate value. This can be very positive ('+++' in the Tables of Section 2.3), positive ('++'), somewhat positive ('+'), neutral ('o'), somewhat negative ('-'), negative ('--'), and very negative ('---')

Furthermore, the following additional terms are used:

Table 35. Terms used to describe aggregates of the patient and therapy

Term	Explanation
has_influence_on_patient_state: MEASUREMENT x TREND x AGGREGATE_VALUE x INFLUENCE	This expresses the influence relation of a certain trend for a measurement upon an aggregate value of the patient state.
has_influence_on_therapeutic_state: MEASUREMENT x TREND x AGGREGATE_VALUE x INFLUENCE	This expresses the influence relation of a certain trend for a measurement upon an aggregate value of the therapeutic state.
has_numeric_value: INFLUENCE x REAL	This expresses how the qualitative influence expression can be changed into a numerical value in order for it to be used to calculate the new value for the patient state
current_patient_state: AGGREGATE_VALUE x REAL	The value for a particular aggregate measurement of the patient state
current_therapeutic_state: AGGREGATE x REAL	The value for a particular aggregate measurement of the therapeutic state

Given the combination of terms expressed above, the following equations can be used to update the current value of the patient. It is assumed that the calculations are performed in a sequential manner.

∀M:MEASUREMENT, T:TREND, A:AGGREGATE_VALUE, I:INFLUENCE, R, S:REAL
has_influence_on_patient_state(M, T, A, I) &
has_numeric_value(I, R) &
current_patient_state(A, S) & $R \geq 0 \rightarrow$
current_patient_state(A, $S + \alpha \cdot (1-S) \cdot R$)

∀M:MEASUREMENT, T:TREND, A:AGGREGATE_VALUE, I:INFLUENCE, R, S:REAL
has_influence_on_patient_state(M, T, A, I) &
has_numeric_value(I, R) &
current_patient_state(A, S) & $R < 0 \rightarrow$
current_patient_state(A, $S + \alpha \cdot S \cdot R$)

The above express the influence of the trends in case of a positive influence (upper equation) and in case of a negative influence (lower equation). The arrow $\rightarrow$ represents a temporal condition, namely that the antecedent should be true for one time point in order for the consequent to become true from 1 time point. In case the contribution is positive, the value for the patient state is adjusted upwards. How much the state is increased depends on the strength of the contribution (R) and the speed factor α. In case the contribution is negative, the value is adjusted in a downward direction. The same principle applies for the judgment of the therapeutic state of the patient:

∀M:MEASUREMENT, T:TREND, A:AGGREGATE_VALUE, I:INFLUENCE, R, S:REAL
has_influence_on_therapeutic_state(M, T, A, I) &
has_numeric_value(I, R) &
current_therapeutic_state(A, S) & R ≥ 0 →
current_therapeutic_state(A, S + α·(1-S)·R)

∀M:MEASUREMENT, T:TREND, A:AGGREGATE_VALUE, I:INFLUENCE, R, S:REAL
has_influence_on_therapeutic_state(M, T, A, I) &
has_numeric_value(I, R) &
current_therapeutic_state(A, S) & R < 0 →
current_therapeutic_state(A, S + α·S·R)

5. Conclusions

Throughout this deliverable, it has been shown what kind of measurements will be performed using the various devices within the ICT4Depression system, and how this information is then aggregated to make a good judgment of the current state of the patient and the therapy. In order to do so, a formal temporal language has been used which has been developed at the VU University Amsterdam. This temporal language allows for the expression of complex temporal properties that can both be expressed in a qualitative and quantitative fashion, making it very suitable for the domain at hand. It has been attempted to make the proposed abstraction approach as generic as possible in order to allow for the approach to be usable in other domains as well. One crucial aspect in the approach is that the parameters need to be set to an appropriate value. For instance, for what size intervals will the trends be identified? And how much numerical influence does a '+++' influence indicated by the psychologists mean? All these parameters will be set to an appropriate value in accordance with the expertise of the experts in the domain of Clinical Psychology involved within the project.

The next step in the research will be to create formal expression of the therapy to enable forecasting of the patient in the future, also given the trends and state that have been derived based upon the methods described in this deliverable.

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